Class 17 in-class problems, 18.05, Spring 2022

Concept questions

Concept question 1. What would a frequentist say?
Each day Jaam arrives X hours late to class, with $X \sim \text{uniform}(0, \theta)$, where θ is unknown. Jon models his initial belief about θ by a prior pdf $f(\theta)$. After Jaam arrives x hours late to the next class, Jon computes the likelihood function $\phi(x|\theta)$ and the posterior pdf $f(\theta|x)$.

Which of these probability computations would the frequentist consider valid?

1. none
2. prior
3. likelihood
4. posterior
5. prior and posterior
6. prior and likelihood
7. likelihood and posterior
8. prior, likelihood and posterior.

Concept question 2. Is it a statistic. Suppose $x_1, \dots, x_n$ is a sample from $\text{N}(\mu, \sigma^2)$, where μ and σ are unknown.

Is each of the following a statistic?

(a) The median of $x_1, \dots, x_n$.

(b) The interval from the 0.25 quantile to the 0.75 quantile of $\text{N}(\mu, \sigma^2)$.

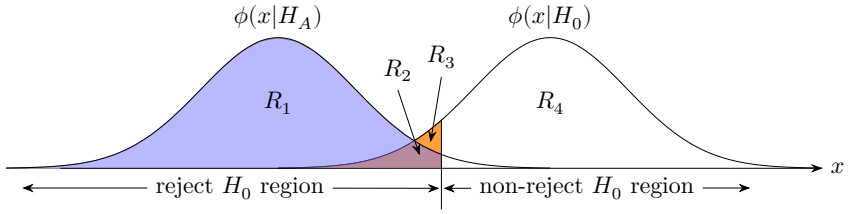
(c) The standardized mean $\dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$.

(d) The set of sample values less than 1 unit from $\bar{x}$.

(e) The $z = \dfrac{\overline{x} - 5}{3/\sqrt{n}}$.

(f) $z = \dfrac{\overline{x} - \mu_0}{\sigma_0/\sqrt{n}}$, where μ_0 and σ_0 are given values,

Concept question 2. Picture the significance. The null and alternate pdfs are shown on the following plot



The significance level of the test is given by the area of which region?

1. R_1
2. R_3
3. $R_1 + R_2$
4. $R_2 + R_3 + R_4$
5. R_2
6. R_4
7. $R_2 + R_3$
8. None of these

Board questions

Problem 1. Testing coins

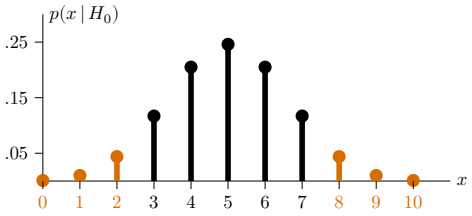
Suppose we have a coin with an unknown probability of heads θ.

Test statistic x = number of heads in 10 tosses.

Null hypothesis H_0: $\theta = 0.5$ (fair coin).

Alternative hypothesis H_A: $\theta \neq 0.5$ (unfair coin, two-sided).

(a) The rejection region is are the values of x shown in orange. What's the significance level?



x	0	1	2	3	4	5	6	7	8	9	10
$p(x\|H_0)$	0.001	0.010	0.044	0.117	0.205	0.246	0.205	0.117	0.044	0.010	0.001

(b) For significance level $\alpha = 0.05$, find a two-sided rejection region.

Problem 2. z statistic

Suppose we know the following about our null hypothesis significance test.

- H_0: data follows a $N(5, 10^2)$
- H_A: data follows a $N(\mu, 10^2)$ where $\mu \neq 5$.
- Test statistic: z = standardized $\overline{x}$.
- Data: 64 data points with $\overline{x} = 6.25$.
- Significance level set to $\alpha = 0.05$.

(a) Find the rejection region; draw a picture.

(b) Find the z-value; add it to your picture.

(c) Decide whether or not to reject H_0 in favor of H_A.

(d) Find the p-value for this data; add to your picture.

(e) What's the connection between the answers to (b), (c) and (d)?

Problem 2. More coins

Two coins: probability of heads is 0.5 for C_1; and 0.6 for C_2.

We pick one at random, flip it 8 times and get 6 heads.

Here are the probability tables for the two coins

k	0	1	2	3	4	5	6	7	8
$p(k\|\theta = 0.5)$	0.004	0.031	0.109	0.219	0.273	0.219	0.109	0.031	0.004
$p(k\|\theta = 0.6)$	0.001	0.008	0.041	0.124	0.232	0.279	0.209	0.090	0.017

(a) H_0 = 'The coin is C_1' H_A = 'The coin is C_2'

Do you reject H_0 at the significance level $\alpha = 0.05$?

(Hint: First decide if this test is two-sided, left-sided or right-sided. Then determine the rejection region.)

(b) H_0 = 'The coin is C_2' H_A = 'The coin is C_1'

Do you reject H_0 at the significance level $\alpha = 0.05$?

(c) Do your answers to (a) and (b) seem paradoxical

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