Class 16 in-class problems, 18.05, Spring 2022

Concept questions

Concept question 1. Increasing probability

To convert an 80% probability interval to a 90% interval should you shrink it or stretch it?

1. *Shrink* **2.** *Stretch.*

Solution: 2. Stretch. A bigger probability requires a bigger interval.

Board questions

Problem 1. Treating severe respiratory failure*

Two treatments for newborns with severe respiratory failure.

1. *CVT: conventional therapy (hyperventilation and drugs)*

2. *ECMO: extracorporeal membrane oxygenation (invasive procedure)*

In 1983 in Michigan:

19/19 ECMO babies survived and 0/3 CVT babies survived.

Later Harvard ran a randomized study:

28/29 ECMO babies survived and 6/10 CVT babies survived.

**Adapted from* Statistics: a Bayesian Perspective *by Donald Berry*

Name the probabilites of survival:

θ_E = *probability that an ECMO baby survives*
θ_C = *probability that a CVT baby survives.*

Consider the values 0.125, 0.375, 0.625, 0.875 for θ_E *and* θ_C.

(a) *Make the* 4×4 *prior table for a flat prior.*

(b) *Based on the Michigan results, create a reasonable informed prior table for analyzing the Harvard results (unnormalized is fine).*

(c) *Make the likelihood table for the Harvard results. (You might use R to compute some of the values.)*

(d) *Find the posterior table for the informed prior.*

(e) *Using the informed posterior, compute the probability that ECMO is better than CVT.*

(f) *Also compute the posterior probability that* $\theta_E - \theta_C \geq 0.6$.

(The posted solutions will also show 4-6 for the flat prior.)

Solution: (a) Flat prior

		θ_E			
		0.125	0.375	0.625	0.875
	0.125	0.0625	0.0625	0.0625	0.0625
θ_C	0.375	0.0625	0.0625	0.0625	0.0625
	0.625	0.0625	0.0625	0.0625	0.0625
	0.875	0.0625	0.0625	0.0625	0.0625

(b) Informed prior (This table is unnormalized)

		θ_E			
		0.125	0.375	0.625	0.875
	0.125	18	18	32	32
θ_C	0.375	18	18	32	32
	0.625	18	18	32	32
	0.875	18	18	32	32

Rationale: Since 19/19 ECMO babies survived we believe θ_E is probably near 1.0. That 0/3 CVT babies survived is not enough data to move from a uniform distribution. (Or we might distribute a little more probability to larger θ_C.) So for θ_E we split 64% of probability in the two higher values and 36% for the lower two. Our prior is the same for each value of θ_C.

(c) Likelihood

Entries in the likelihood table: $c\theta_E^{28}(1-\theta_E)\theta_C^6(1-\theta_C)^4$. The constant c is computed from binomial coefficients. It is unimportant for updating. The table was computed using R.

		θ_E			
		0.125	0.375	0.625	0.875
	0.125	6.160e-28	1.007e-14	9.835e-09	4.048e-05
θ_C	0.375	1.169e-25	1.910e-12	1.866e-06	7.682e-03
	0.625	3.247e-25	5.306e-12	5.184e-06	2.134e-02
	0.875	3.019e-26	4.932e-13	4.819e-07	1.984e-03

Extra: flat posterior

The posterior table is found by multiplying the prior and likelihood tables and normalizing so that the sum of the entries is 1. We call the posterior derived from the flat prior the flat posterior. (Of course the flat posterior is not itself flat.)

		θ_E			
		0.125	0.375	0.625	0.875
	0.125	1.984e-26	3.242e-13	3.167e-07	0.001
θ_c	0.375	3.765e-24	6.152e-11	6.011e-05	$\boxed{0.247}$
	0.625	1.046e-23	1.709e-10	1.670e-04	$\boxed{0.687}$
	0.875	9.721e-25	1.588e-11	1.552e-05	0.0639

The boxed entries represent most of the probability where $\theta_E > \theta_C$.

All our computations were done in R. For the flat posterior:

Probability ECMO is better than CVT is

$$P(\theta_E > \theta_C \,|\, \text{Harvard data}) = 0.936$$
$$P(\theta_E - \theta_C \geq 0.6 \,|\, \text{Harvard data}) = 0.001$$

(d) Informed posterior

		θ_E			
		0.125	0.375	0.625	0.875
	0.125	1.116e-26	1.823e-13	3.167e-07	0.001
θ_C	0.375	2.117e-24	3.460e-11	6.010e-05	$\boxed{0.2473}$
	0.625	5.882e-24	9.612e-11	1.669e-04	$\boxed{0.6871}$
	0.875	5.468e-25	8.935e-12	1.552e-05	0.0638

For the informed posterior:

$$P(\theta_E > \theta_C \,|\, \text{Harvard data}) = 0.936$$
$$P(\theta_E - \theta_C \geq 0.6 \,|\, \text{Harvard data}) = 0.001$$

Note: Since both flat and informed prior gave essentially the same answers, we gain confidence that these calculations are robust. That is, they are not too sensitive to our exact choice of prior.