Class 12 in-class problems, 18.05, Spring 2022

Concept questions

Concept question 1. Three coins

- Type $C_{0.5}$ coins are fair, with probability 0.5 of heads
- Type $C_{0.6}$ coins have probability 0.6 of heads
- Type $C_{0.9}$ coins have probability 0.9 of heads

A drawer has one of each.

Pick one; toss 5 times; Suppose you get 1 head out of 5 tosses.

What's your best guess for the probability of heads on the next toss?

(a) 0.1 **(b)** 0.2 **(c)** 0.3 **(d)** 0.4

(e) 0.5 **(f)** 0.6 **(g)** 0.7 **(h)** 0.8

(i) 0.9 **(j)** 1.0

Concept question 2. Suppose instead of saying 1 head in 4 tails, we told you the tosses, in order, were THTTT. Does this affect posterior distribution of the coin type?

(a) Yes **(b)** No

Board questions

Problem 1. Three coins

- We have 3 coins with probabilities 0.5, 0.6, and 0.9 of heads.
- Pick one at random; toss 5 times.
- Suppose you get 1 head out of 5 tosses.

Compute the posterior probabilities for the type of coin and the posterior predictive probabilities for the results of the next toss.

(a) Specify clearly the set of hypotheses and the prior probabilities.

(b) Compute the prior and posterior predictive distributions, i.e. give the probabilities of all possible outcomes.

Problem 2. Screening test odds

A disease is present in 0.005 of the population.

A screening test has a 0.05 false positive rate and a 0.02 false negative rate.

(a) Give the prior odds a patient has the disease

Assume the patient tests positive

(b) What is the Bayes factor for this data?

(c) What are the posterior odds they have the disease?

(d) Based on your answers to (a) and (b) would you say a positive test (the data) provides strong or weak evidence for the presence of the disease.

Problem 3. CSI Blood Types
We need to weigh the evidence at a crime scene:

- Crime scene: the two perpetrators left blood: one of type O and one of type AB
- In population 60% are type O and 1% are type AB

(a) Suspect Oliver is tested and has type O blood.

Compute the Bayes factor and posterior odds that Oliver was one of the perpetrators.

Is the data evidence for or against the hypothesis that Oliver is guilty?

(b) Same question for suspect Alberto who has type AB blood.

Hopefully helpful hints:

For the question about Oliver we have

Hypotheses:
S = 'Oliver and another unknown person were at the scene'
S^c = 'two unknown people were at the scene'

Data: D = 'type 'O' and 'AB' blood were found; Oliver is type O'

Prior odds: These are unknown. We can just say $O(S)$.

*From 'Information Theory, Inference, and Learning Algorithms' by David J. C. Mackay.

18.05 Introduction to Probability and Statistics
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