

Part II Problems and Solutions

Problem 1: Find the general real solution of

$$\frac{d^3x}{dt^3} - \frac{dx}{dt} = t^2 + 1$$

Solution: characteristic polynomial $p(s) = s^3 - s$, $p(0) = 0$ and so we can't apply undetermined coefficients directly. Let $u = \dot{x}$, so $\ddot{u} - u = t^2 + 1$. Try $u = at^2 + bt + c$, so $\ddot{u} = 2a$ and $t^2 + 1 = \ddot{u} - u = -at^2 - bt + (2a - c)$ implies $a = -1$, $b = 0$, $2a - c = 1$ or $c = -3$: so $u_p = -t^2 - 3$. Then x_p is the integral of u_p :

$$x_p = -\frac{1}{3}t^3 - 3t.$$

To solve the homogeneous equation, factor $p(s) = s(s-1)(s+1)$ so $x_h = c_1 + c_2e^t + c_3e^{-t}$. General solution: $x = x_p + x_h$.

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