Part II Problems

Problem 1: [Models and complex gain] This problem employs the Mathlet `Amplitude and Phase: Second Order I`, which illustrates the steady state sinusoidal system response of a spring/mass/dashpot system driven through the spring, as discussed in the session on Gain and Phase Lag. The amplitude of the input signal is 1, so the amplitude of the system response is the gain. Select $b = 0.5$, $k = 4.00$, and $\omega = 2.00$. Animate the system.

(a) Verify the displayed values of time lag and gain.

(b) The input signal in this system is the position $y(t) = \cos(\omega t)$ of the top of the spring (the cyan box). The system response is the position of the mass (the yellow box), and we are looking just at the steady state solution $x(t)$. Determine the complex gain for this system. From it determine the gain of the system, as a function of ω. Finally determine $\tan\phi$ where ϕ is the phase lag.

Problem 2: [Frequency response] This problem will use the applet `Amplitude and Phase: Second Order I` again (as in a previous problem). Set $k = 4.00$, $b = 0.50$. These settings will be in force for parts **(a)** through **(c)**.

(a) In a previous problem you studied the response of this system when $\omega = 2.00$. The gain is pretty large with that setting. Let's investigate the gain for other values of ω. You can see a graph of the gain as a function of the input circular frequency ω by invoking `[Bode Plots]`. The top window shows the gain as a function of ω, and the bottom window graphs $-\phi$ as a function of ω. Move the ω slider and verify that these readings correspond to the graph of the system response at left. You can see a readout of the value of the gain and the phase lag for the selected value of ω by rolling the cursor over the relevant window.

From your experimentation with the applet, do you believe that the gain maximal for $\omega = 2.00$, or is the "practical resonance" peak at a different value of ω?

In the previous problem mentioned you wrote down a formula for the gain as a function of ω (for these values of k and b), $g(\omega)$. Now find the value $\omega = \omega_r$ which maximizes $g(\omega)$. (Hint: it'll be easier to minimize the square of the denominator.) Is it at $\omega = 2$? Finally, what is the maximal gain?

(b) Experiment to find the value of ω giving phase lag as close to $45°$ as you can. In previous problem mentioned you also gave a formula for $\tan\phi$. Determine the positive value of ω for which the phase lag equals $45°$. Compare.

(c) Now invoke the `[Nyquist Plot]`. This shows the trajectory of the complex gain $H(\omega)$ as ω runs from 0 to ∞. The value of $H(\omega)$ corresponding to the selected value of ω is shown as a yellow diamond. This means that the length of the yellow strut equals the gain, and the size of the green arc equals the phase lag. Again grab the ω slider and move it slowly from 0 to 4. Please submit a sketch of the Nyquist plot with ω such that $\phi(\omega)$ is as close to $\frac{\pi}{4}$ as you can get it.

(d) Finally, set $\omega = 2$ and leave $k = 4.00$, but adjust the value of b by grabbing the b slider. What do you observe about the position of the yellow strut in the Nyquist plot? Try setting k to a different value, and adjust ω so that the phase lag is close to $\frac{\pi}{2}$. Now vary b and comment on what happens to the phase lag. Please explain this observation as follows. Write down a formula for the complex gain $H(\omega)$ for general values of b, k, and ω. What does $\phi = \frac{\pi}{2}$ say about the complex gain? Finally, what relationship does this imply about b, k, and ω? Does this relationship explain your observation?