

## 18.03SC Practice Problems 28

### Inverse Laplace transform

#### Rules for the Laplace transform

Definition:  $\mathcal{L}[f(t)] = F(s) = \int_{0^-}^{\infty} f(t)e^{-st} dt$  for  $\text{Re}(s) \gg 0$ .

Linearity:  $\mathcal{L}[af(t) + bg(t)] = aF(s) + bG(s)$ .

$\mathcal{L}^{-1}$ :  $F(s)$  essentially determines  $f(t)$  for  $t > 0$ .

$s$ -shift rule:  $\mathcal{L}[e^{rt}f(t)] = F(s - r)$ .

$s$ -derivative rule:  $\mathcal{L}[tf(t)] = -F'(s)$ .

$t$ -derivative rule:  $\mathcal{L}[f'(t)] = sF(s) - f(0^-)$ .

#### Formulas for the Laplace transform

$$\mathcal{L}[1] = \frac{1}{s}, \quad \mathcal{L}[\delta(t - a)] = e^{-as}$$

$$\mathcal{L}[e^{rt}] = \frac{1}{s - r}, \quad \mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}[\cos(\omega t)] = \frac{s}{s^2 + \omega^2}, \quad \mathcal{L}[\sin(\omega t)] = \frac{\omega}{s^2 + \omega^2}$$

1. Find the inverse Laplace transform for each of the following.

$$\frac{2s + 1}{s^2 + 9}, \quad \frac{s^2 + 2}{s^3 - s}, \quad \frac{2}{s^2(s - 1)}$$

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18.03SC Differential Equations  
Fall 2011

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