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18.034 Honors Differential Equations
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18.034 Midterm #2 Name:

1. (a) (10 points) Find a linear differential equation with constant coefficients that has solutions t, e^t and te^t.

(b) (10 points) Prove or disprove that t^4 and t^6 can be solutions of one and the same linear homogeneous differential equation $y'' + p(t)y' + q(t)y = 0$ on an interval $[-1, 1]$, where p and q are continuous on $[-1, 1]$.

2. For a certain regular linear differential operator L a basis of solutions of $Ly = 0$ is given by

$$y_1(t) = e^{2t}, \qquad y_2(t) = 2t^2 + 2t + 1.$$

(a) (10 points) Compute the Wronskian of y_1 and y_2.

(b) (10 points) Using *variation of parameters*, find a solution of the differential equation $Ly = t^2 e^{2t}$.

3. (a) (10 points) If the constant a is not a root of the polynomial p with real coefficients, show that a particular solution of $p(D)y = e^{at}$ is

$$y(t) = \frac{e^{at}}{p(a)}.$$

(b) (10 points) Find a particular solution of

$$(D^4 + D^3 + D^2 + D + 1)y = 33e^{-2t}.$$

4. (20 points) Find the general solution of

$$y''' + 3y'' + 3y' + y = te^{-t}.$$

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5. Consider the initial value problem

$$y' = 1 + y^2, \qquad y(0) = 0.$$

(a) (10 points) Using Picard's iteration method obtain the iterates $y_1(t)$ and $y_2(t)$.

(b) (10 points) Show that the initial value problem has at most one solution in any interval of the form $t \in (-a, a)$.

(c) (extra credits) Find the exact solution $y(t)$ and show that $\lim_{n\to\infty} y_n(t) = y(t)$.