18.034, Honors Differential Equations
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**Rec. Suggestions.**
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1. Do 1 or 2 more IVP's by Laplace transform including computation of partial fractions, e.g

$$\begin{cases} y''+y = \cos(2t) \\ y(0) = 0 \\ y'(0) = 1 \end{cases} \rightsquigarrow \quad \underline{Y}(s) = \frac{1}{s^2+1} + \frac{1}{3}\frac{s}{s^2+1} - \frac{1}{3}\frac{s}{s^2+4}$$

$$y(t) = \sin(t) + \frac{1}{3}\cos(2t) - \frac{1}{3}\cos(2t).$$

2. Use periodic function rule to deduce,

$$\int_0^{2\pi} e^{-st}\cos(t)dt = \frac{s}{s^2+1}\left(1-e^{-2\pi s}\right), \qquad \int_0^{2\pi} e^{-st}\sin(t)dt = \frac{1}{s^2+1}\left(1-e^{-2\pi s}\right)$$

You might mention that taking the power series in $s$ of each side gives closed formulas,

e.g. $$\int_0^{2\pi} t^{k+1}\sin(t)dt = (k+1)!\sum_{l=0}^{\left[\frac{z}{\upsilon}\right]} \frac{(-1)^{l+1}(2\pi)^{k+1-2l}}{(k+1-2l)!}$$

This can be used to compute Fourier coefficients of $t^{k+1}$.

3. Use L to prove $S(t)t^a * S(t)t^b = \frac{a!\,b!}{(a+b+1)!}S(t)\,t^{a+b+1}$

$S(t) * \cos(t) = S(t)\sin(t)$

4. Consider $p(s) = s^2 + As - B$. In the 3 cases that

(i) $p(s) = (s - r_1)(s - r_2)$, (ii) $p(s) = (s - r)^2$, (iii) $p(s) = (s - \alpha)^2 + \beta^2$,

compute $L^{-1}\left[\frac{1}{p(s)}\right]$. (i.e. first row of Table 5.4.1).