

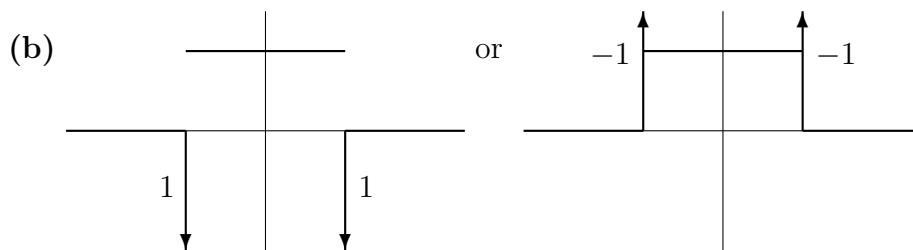
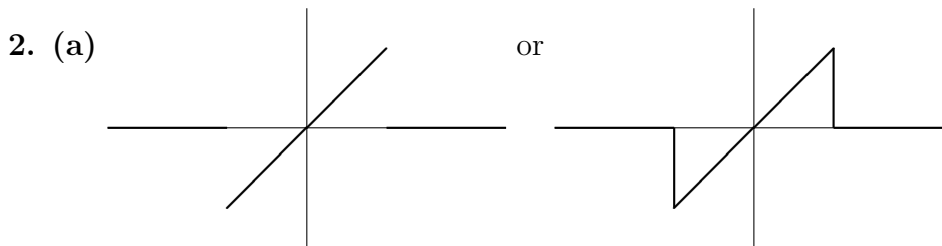
## 18.03 Hour Exam III Solutions: April 23, 2010

1. (a) The minimal period is 2.

(b)  $f(t)$  is even.

$$(c) x_p(t) = \frac{1}{\omega_n^2} + \frac{\cos(\pi t)}{2(\omega_n^2 - \pi^2)} + \frac{\cos(2\pi t)}{4(\omega_n^2 - 4\pi^2)} + \frac{\cos(3\pi t)}{8(\omega_n^2 - 9\pi^2)} + \dots$$

(d) There is no periodic solution when  $\omega_n = 0, \pi, 2\pi, 3\pi, \dots$



(c)  $f'(t) = (u(t+1) - u(t-1)) - \delta(t+1) - \delta(t-1)$ ;  $f'_r(t) = u(t+1) - u(t-1)$ ,  
 $f'_s(t) = -\delta(t+1) - \delta(t-1)$ .

$$3. (a) v(t) = w(t) * u(t) = \int_0^t w(t-\tau)u(\tau) d\tau = \int_0^t (e^{-(t-\tau)} - e^{-3(t-\tau)}) d\tau$$

$$= e^{-t} e^\tau \Big|_0^t - e^{-3t} \frac{e^{3\tau}}{3} \Big|_0^t = (1 - e^{-t}) - \frac{1 - e^{-3t}}{3} = \frac{2}{3} - e^{-t} + \frac{e^{-3t}}{3}.$$

$$(b) W(s) = \mathcal{L}[w(t)] = \frac{1}{s+1} - \frac{1}{s+3}.$$

$$(c) W(s) = \frac{(s+3) - (s+1)}{(s+1)(s+3)} = \frac{2}{s^2 + 4s + 3}, \text{ so } p(s) = \frac{1}{2}(s^2 + 4s + 3).$$

$$4. (a) \frac{s-1}{s} = 1 - \frac{1}{s} \rightsquigarrow \delta(t) - u(t), \text{ so } \frac{e^{-s}(s-1)}{s} \rightsquigarrow \delta(t-1) - u(t-1).$$

(b)  $F(s) = \frac{s+10}{s^3 + 2s^2 + 10s} = \frac{a}{s} + \frac{b(s+1)+c}{(s+1)^2 + 9}$ . By coverup,  $a = \frac{10}{10} = 1$ . By complex coverup (multiply through by  $(s+1)^2 + 9$  and set  $s$  to be a root, say  $-1+3i$ ),  $b(3i)+c = \frac{9+3i}{-1+3i} = -3i$ , so  $b = -1$ ,  $c = 0$ , and  $F(s) = \frac{1}{s} - \frac{s+1}{(s+1)^2 + 9}$ , which is the Laplace transform of  $1 - e^{-t} \cos(3t)$ .

5. (a)  $\{0, -1+3i, -1-3i\}$ .

$$(b) X(s) = W(s)F(s). F(s) = \frac{2}{s^2 + 4}, \text{ so } X(s) = \left( \frac{s+10}{s^3 + 2s^2 + 10s} \right) \left( \frac{2}{s^2 + 4} \right).$$

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18.03 Differential Equations  
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