Vector derivatives

1. Let $\mathbf{r}(t)$ be a vector function. Prove by using components that

$$\frac{d\mathbf{r}}{dt} = \mathbf{0} \;\Rightarrow\; \mathbf{r}(t) = \mathbf{K}, \text{ where } \mathbf{K} \text{ is a constant vector.}$$

<u>**Answer:**</u> In two dimensions $\mathbf{r}(t) = \langle x(t), y(t)\rangle, \mathbf{r}'(t) = \langle x'(t), y'(t)\rangle$.
Therefore,

$$\begin{aligned}
\mathbf{r}'(t) = 0 \quad &\Rightarrow \quad x'(t) = 0 \text{ and } y'(t) = 0 \\
&\Rightarrow \quad x(t) = k_1 \text{ and } y(t) = k_2 \\
&\Rightarrow \quad \mathbf{r}(t) = \langle k_1, k_2\rangle, \text{ where } k_1 \text{ and } k_2 \text{ are contants.}
\end{aligned}$$

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