**CHRISTINE BREINER:** Welcome back to recitation. In this video, I really just want to practice matrix multiplication, which is potentially something new for some of you, and maybe some of you have been doing it for a while and are very good at it. But I want to make sure that everyone is feeling confident in their ability to multiply matrices.

So we have three matrices here. We have A, B, and C. And what I want you to do is I want you to compute what makes sense below. I have four products of matrices below.

a is A times B, b is B times A, c is B times C, and d is A times C. So I want you to multiply the matrices that make sense to multiply, and then the ones that don't, make sure you understand why. Give yourself a brief explanation of why you can't multiply them.

So why don't you work on that, pause the video, and when you feel confident in your answers, bring the video back up, and you can check them against my work.

OK, welcome back. Well, we wanted to make sure we felt comfortable multiplying matrices. So what we're going to do is look at the four products I mentioned below, and we're going to see how they do, whether or not we can actually compute them.

So let's first look at a, which was A times B. So before I write it down again, A times B is-- notice A is a 2 by 2 matrix, and B is a 2 by 3 matrix, right?

And so if I write letter a, we know we're taking a 2 by 2 by a 2 by 3. And so the fact that the interior dimensions agree, that the number of columns of A is equal to the number of rows of B, means that I can multiply them. So I can multiply them, and my result I expect to get is, of course, the dimensions we have on the outside. So I expect to get a 2 by 3 matrix.

So I'm going to rewrite A and B here, so that I don't have to keep walking back and forth, and then we'll do the multiplication. So I have [6, 5; 1, 2] times [2, -1, 3; 1, 0, 4]. OK. So I want to perform this multiplication.

Now remember that when you are looking for a value in your resulting matrix, which I know is 2 by 3, so I can even make a little space for myself. I know it's 2 by 3, so I know I'm going to have to fill in these spots.

When I look at this position, it's row 1, column 1. That means I take row 1 of the first matrix, and I'm essentially just dotting it with column 1 of the second matrix. So I'm taking row 1 times column 1 in the way it was described, which is I take 6 times 2, and I add it to 5 times 1. So row 1, column 1 gives me 6 times 2 is 12, plus 5 times 1 which is 5, so I get 17.

And then if I come in to the next spot, what is this? And the resulting matrix's position is row 1, column 2. So now I take row 1 of the first matrix and column 2 of the second matrix, and I get 6 times negative 1 is negative 6, plus 0 times 5, so I get a negative 6 here. Negative 6 times 0. Maybe I should show you this way. Negative 6 times 0. OK.

And then here I am now in the third spot of the first row, so I'm in a row 1, column 3. So that's again, row 1 of the first, column 3 of the second. So you see a pattern here about where we're getting our things from that we're multiplying. For row 1, column 3 of the resulting, I take row 1 of the first and column 3 of the second.

So 6 times 3 is 18, plus-- 5 times 4 is 20. So 20 plus 18 is 38. OK.

Now I have to do the same thing on the bottom. Right? So I have now here, the row, notice I'm in row 2, so I'm always going to use row 2 of this first matrix. And then what we saw last time is I used column 1 in the first spot, column 2 in the second spot, column 3 in the third spot. Right? That's what happens over and over again.

So what do I do? I take [1, 2], and I multiply it by [2, 1]. So I take 1 times 2, plus 2 times 1. So I get 2 there and 2 there, so I get a 4.

And then the next column: row 2, column 2. 1 times negative 1 is negative 1-- 2 times 0 is 0-- so I get a negative 1.

And then the last column. 1 times 3 is 3. 2 times 4 is 8. So I get 3 plus 8, so I get 11.

Hopefully I didn't make any stupid summing mistakes there, but if I did, you probably caught it. Because I was trying to say what I did as we went. So that is the answer to a.

OK. So now let's think about what is b. b was take B times A, which is just to switch the order. So let's look at the dimension match-up.

Now we have a 2 by 3 matrix, and I'm trying to multiply it by a 2 by 2 matrix. Well, I don't have to do any more work, because I can't do it. Because the dimensions of the insides here-- three columns for B, two rows for A-- means that I can't actually multiply them. OK? So this isn't even defined. OK, so that was easy. That was b. All right.

Letter c-- I'll give myself a lot of room to do that-- letter c was B times C. And so I'm going to write down the dimensions to see if I even need to write down the matrices. B was two rows by three columns, and C was three rows by two columns. So if I look at the dimensions, the 3 and the 3 match up, so I am going to be able to multiply them, and my result-- as I mentioned before-- should be a 2 by 2.

So let me write down B and C here, so we don't have to keep going to the side. OK. And then C is 1, 2 negative 1; negative 1, 3, 2. All right, let me just make sure I didn't transcribe anything incorrectly. I think it looks good. OK.

So row 1 of B. Row 2 of B. Column 1 of C. Column 2 of C. We're going to be dealing with those, specifically. So we want to multiply these. We said our resulting matrix is going to be 2 by 2. OK.

Because I'm going to have a lot of terms, I might write them down on this one, and then simplify. Because I may make a mistake, so to be more careful, I'll write down all the pieces.

So here I am in row 1, column 1 of the resulting. So I take row 1 of the first, column 1 of the second, and what do I get when I multiply? I get 2 times 1-- that's 2. Plus negative 1 times 2-- that's negative 2. Plus 3 times negative 1-- that's negative 3. Right? That's all we have to do for the first position.

Then I do, for the second one, it's row 1, column 2. So I do row 1, column 2. So I'll try to keep my head out of the way. I realize I keep stepping in front. So it's 2 times negative 1. I get negative 2. Negative 1 times 3, so I get negative 3. And 3 times 2 gives me 6. OK. And then the bottom two. I get row 2, column 1 over here, and then row 2, column 2 over here.

So row 2, column 1 is going to be 1 times 1. Plus 0 times 2. Plus 4 times negative 1, so I get negative 4.

And then here. Row 2, column 2. I get 1 times negative 1, so I have negative 1. Plus 0 times 3-- plus 0. And then 4 times 2 is 8.

So if I simplify these, it looks like in the first spot I should get a negative 3. And the second spot, I should get a 1. This is just for you to check your answer. And the third spot, I get a negative 3. And then the fourth spot, I get a 7.

So hopefully I added correctly all throughout. I think I did, so I think we're good there. So that is the answer to c.

And again, the reason we can multiply those, was that the dimensions-- when you wrote them down in order-- the dimensions to the inside agreed, and then the outside gives us the size of the resulting matrix. So there was one more problem, and that was d.

And I wanted you to take A times C. And A was a 2 by 2. And C was a 3 by 2. And so again, we see we can't do it, because the two interior dimensions here-- when I write them in that order-- don't agree. OK. So d is not defined.

All right, so the basic idea of this whole video is just to make sure we felt comfortable multiplying matrices. We're trying to use some simple examples to understand that. Understand how we can recognize from the dimensions whether or not multiplication is even defined, and then what size the resulting matrix will be.

I do want to point out one thing. And I want to point out that if we come over to our example back in the beginning. We had A*B as our first example and then B*A as our second example. And A*B-- well, I think, I got to remember what they were-- yeah, A*B you could multiply, but B*A you could not.

So I think it has been stressed before, but I think I should stress it again, that order matters in multiplication. OK? You can't commute these things. You can't switch the order and get the same result. So matrix multiplication, you have to be very careful about keeping things in the same order as you're multiplying.

OK, I think that is where I will stop.