Problems: Flux Through Surfaces

Let $\mathbf{F} = \langle x, y, z \rangle$.

1. Find the flux of $\mathbf{F}$ through the square with vertices $(0,0,0)$, $(1,0,0)$, $(1,1,0)$, $(0,1,0)$.

Answer: The square in question lies in the plane $z = 0$, so $\mathbf{n} = \langle 0, 0, 1 \rangle$. $\mathbf{F} \cdot \mathbf{n} = z = 0$ on the whole square, so the flux is zero.

2. Find the flux of $\mathbf{F}$ through the square with vertices $(0,0,1)$, $(1,0,1)$, $(1,1,1)$, $(0,1,1)$.

Answer: Again $\mathbf{n} = \langle 0, 0, 1 \rangle$ and $\mathbf{F} \cdot \mathbf{n} = z$.

$$\text{Flux} = \iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \int_0^1 \int_0^1 1\, dx\, dy = 1.$$

3. Find the flux of $\mathbf{F}$ through the surface $x^2 + y^2 = 1$ with $0 \le z \le 1$.

Answer: Here $\mathbf{n} = \langle x, y, 0 \rangle$, so $\mathbf{F} \cdot \mathbf{n} = x^2 + y^2 = 1$. We can parametrize the surface by $x = \cos\theta$, $y = \sin\theta$ with $dS = d\theta\, dz$ and integrate, or we can observe that the result of that calculation will just be the surface area of the cylinder. Flux $= 2\pi$.