18.02 Problem Set 5

Due Thursday 3/16/06, 12:55 pm

Part A   (10 points)

Hand in the <u>underlined</u> problems only; the others are for more practice.

Lecture 13. Thurs, Mar 9. Lagrange multipliers
    Read: 19.8
    Work: 2I/ <u>1</u>, <u>3</u>.

Lecture 14. Fri, Mar 10. Nonindependent variables.
    Read: N (p. 0–5).   Work: 2J/ <u>1</u>, <u>2</u>, 3a<u>b</u>, 4a<u>b</u>, <u>5a</u>, 6, <u>7</u>.

Lecture 15. Tues, Mar 14. Partial Differential Equations. Review.
    Read: P.   Work: 2K/ 1, 2, 3, <u>4</u>, <u>5</u> (birth of Fourier analysis).

Lecture 16. Thurs, Mar 16. **Exam 2 , covering Lectures 8–15**

Part B   (16 points)

Directions: Attempt to solve *each part* of each problem yourself. If you collaborate, solutions must be written up independently. It is illegal to consult materials from previous semesters. With each problem is the day it can be done.

Problem 0. (not until due date; 2 points)

Write the names of all the people you consulted or with whom you collaborated and the resources you used, or say "none" or "no consultation".

Problem 1. (Thursday, 6 points: $2 + 4$)

a) We seek a more precise answer to PS4/Problem 6b. Use the method of Lagrange multipliers to write down the system of equations satisfied by the point closest to $(2, 1, 10)$ at which $z - 2x^2 - y^2 = 0$

b) Solve the equations to get a numerical approximation to four decimal places for the location of the point. Compare your answers with the approximation in 6b. Was each coordinate of the approximate answer in 6b within $1/100$ of the exact answer? (Suggestion: Express x, y, z as a function of the Lagrange multiplier λ, then substitute these formulas into the constraint equation. After clearing the denominator, the constraint equation leads to the quintic equation $4\lambda^5 + 92\lambda^4 + 253\lambda^3 + 242\lambda^2 + a_1\lambda + 2 = 0$ — you find a_1. Use a calculator to solve this equation. Alternatively, in Matlab, roots($[4, 92, 253, 242, a_1, 2]$) finds the roots. Decide which root is the right one.)

Problem 2. (Friday, 8 points: $4 + 2 + 2$)

a) Suppose that $g(x, y) = c$ a constant and $w = f(x, y, z)$. Which of the following makes sense as the derivative $\partial w/\partial x$? (If so, compute it in terms of the formal derivatives f_x, f_y, f_z, g_x, and g_y. If not, explain why not.)

   i) $\left(\dfrac{\partial w}{\partial x}\right)_x$ ii) $\left(\dfrac{\partial w}{\partial x}\right)_y$ iii) $\left(\dfrac{\partial w}{\partial x}\right)_z$

b) Suppose that $\sin(x + y) = 4t$ and $w = xyt - 2$. Find $(\partial w/\partial t)_x$.

c) Consider the curve of points (x, y, z) satisfying $x^3 - zyx = -7$ and $x - y^2 z + z^3 = 49$. Use the method of total differentials to find dz/dy at $(x, y, z) = (1, 2, 4)$.