CHRISTINE BREINER: Welcome back to recitation. In this video, I'd like us to practice integration by parts. Specifically, I'd like to solve the following four problems. Or I'd like you to solve the following four problems. I'd like us to find antiderivatives for each of these functions. x e to the minus x, x cubed over the quantity 1 plus x squared squared, arctan x, and natural log x over x squared.

And so the main goal, because we're using integration by parts, is to figure out what you should make u, and what you should make v prime. And why don't you give it a shot. Work on that for a little bit.

I'm actually going to give you one hint, and that's that this one, you may want to break up in a nontraditional way. You may not want to break it up as x cubed and 1 over this function. You're going to want to split up this function in the numerator. Part of it will be in u, part of it will be in v prime. So that's my hint on number 2.

So now with that information, I'd like you to give it a shot, and then I'll come back, and I'll show you how I do it.

OK, welcome back. So again, what we're looking for is antiderivatives for each of these four functions. And now, what I'm going to do, is I'm going to help you pick u and v prime, and then I'm going to show you what answer I got. And I'm going to let you do the work in the middle.

So let's start off with number 1. So if I have x e to the minus x-- integral of x e to the minus x dx, it's very easy to do either-- to make either e to the minus x either u or v prime. Doesn't really matter. Because an integral of e to the minus x is going to have an e to the minus x again, and a derivative is going to have an e to the minus x again, with a minus sign in front, in both cases.

But this doesn't really change. So we have-- when we go up or down, it doesn't really matter if we integrate up or take a derivative. So really it's, we get to pick what we do with the e to the minus x based on what we want to do with the x.

Well, we like taking derivatives of things that don't have two functions of x, so it would be nice if we chose our integration by parts pieces so that this thing wasn't there anymore. So let me write down-- actually, before I even do number 1, maybe I should remind you what the integration by parts formula is. So let me just, I'll scratch that out for a second. And what we're doing, is we're going to have integral of u v prime dx. And if you recall what you saw in lecture, is this should be equal to u*v minus the integral v u prime dx. And we'll put that plus c, because sometimes I forget to write it at the end. So I'll put it there, so I don't forget.

So really, what we're trying to do, right, is pick the u and v prime. And so we want to make this thing, this v u prime, as simple as possible. So what I was saying is if we make this v prime or this u, it doesn't matter. So let's pick whether we want this to be u or v prime. Well, if I make this u, then u prime is 1. That's good. If I make it v prime, then v is x squared over 2. That's more complicated. So we obviously want to make this u.

So for number 1, we're going to choose u is equal to x, and v prime is equal to e to the minus x. And then you can proceed from there. And I'll leave it at that. Well, actually, just to make sure we're OK, I'll even write u prime is equal to 1, and v is going to be equal to negative e to the minus x. So we'd be able to proceed from there, right? We have all the pieces we need.

Now, number 2-- I'll give you the final answers at the end. Number 2, picking u and v prime is a little more complicated. And let's look at this function. x cubed over 1 plus x squared squared. The problem with picking-- that does not look like a 2. Sorry.

The problem with picking u and v prime here, is that it's hard to see what's going to be easy to integrate. So what we want to do is rewrite this as-- let's see-- x squared times x over 1 plus x squared squared.

And now, why is this any better? Well, I mean, it's the same thing. But why does this help us see what we want to do? Well, if you notice this thing right here-- 1 plus x squared. What is its derivative? Its derivative is 2x. Up here we have an x. So this piece right here looks like it could be much more easily integrated than this right here.

So this might be a little counterintuitive, because we're going to take the harder-looking thing, and make that our v prime. But the nice thing is that we can actually integrate this quantity. So we choose, in this case, this is our u, and this is our v prime.

So how do I integrate this? Well, I integrate this by using a substitution. And that will give me v. And the derivative of this is quite simple. It's just 2x. Right? But this is the strategy that we want here. Why did we even think to split that up like that? Well, we knew we had to deal with the denominator in some fashion, and taking a derivative with this in the denominator-- so putting this part of the function in u-- when I look at u prime, it's going to be even worse. It's going to be a higher power here. It's going to be a cubic in the denominator. That's just making things worse.

So we know we'd like to integrate this denominator. We'd like it to be a part of v prime. But the problem is that if I put all the x cubed in the u, and if I just had a 1 here for my v prime, that's, I can't really integrate that very well. But if I keep one of the x's, then I can integrate this quite simply with a substitution. So that's the sort of reasoning behind why we choose it that way.

All right. We've got two more to look at, and then I'll give you the answers.

3. OK. 3, you've seen this trick before. The function was arctan x. Now, you've seen this trick I'm about to do with natural log of x. The same kind of thing with natural log of x. You actually saw this in one of the lecture videos. Because there's only one function here, you might think, well, I have no idea what I'm supposed to pick for u and v prime. But remember, it's really arctan x times 1. Now I have two functions.

And what gives us a hint for why we would want to do this, is that what's the derivative of arctan x? Let me just remind you. d/dx of arctan is 1 over 1 plus x squared. Right?

We're back to actually an almost similar situation to what we had in the previous thing. d/dx of arctan x is 1 over 1 plus x squared. So taking a derivative of this puts it in a form that almost looks easy to integrate. What would make this function easy to integrate? If there was an x up here, instead of a 1. Then I could use substitution.

Where do we get that x from when we're solving this problem, where we're actually finding an antiderivative of arctan x? Well, it's going to come from the fact that I make this u, and I make 1 v prime. So let me write that out explicitly. u I make arctan x, and v prime I make 1. What does that do in our formula? Well, we're going to be integrating something that is v u prime. Well, v is going to be x, and u prime we see right here. So it's going to be, I'm going to be integrating x over 1 plus x squared when I started doing the integration by parts method. That's much simpler, as we talked about previously, because the derivative of x squared is 2x, and you have an x in the numerator when you put in that v.

So this is sort of the flavor of how these things are actually working. So let me do the final one here. We have ln x over x squared.

OK. Let me just tell you right now. In integration by parts, natural log x is not something you want to make the v prime. You don't want to try and take an antiderivative. You know an antiderivative of natural log of x. x ln x minus x. But that's certainly not going to make things any easier. Right? You're actually-- then you've got a product of two functions all of a sudden. Everything's getting more complicated.

But natural log of x has a very nice derivative, because you end up with something that has just a power of x. Derivative of natural log of x is just 1 over x. So that's probably the way you always want to go when you see natural log of x in these integration by parts techniques. Because if I choose u is equal to ln x, and then v prime. In this case, I'm going to write it as a power.

Let's think about what happened. u prime is 1 over x, right? So u prime is x to the minus 1. What's v? Well, it's something like, let's see. Negative x to the minus 1. Something like that, right? Let's make sure I did that right. Yeah, I think I did that right.

So all of a sudden, if I integrate v u prime, that's just a power rule. It's x to the minus 2, negative x to the minus 2. So that's quite easy to integrate.

So again, when I see natural log of x in an integration by parts method, almost always, I hate to say always, almost always, almost a guarantee that you want to take a derivative. You want to make that the u.

So hopefully that makes sense, some of these strategies. I tried to pick ones that were somewhat different, so you could see some different types of strategies we needed. And now I've done these earlier. So I'm just going to write down what the answers actually are, and you can compare to what you got. So the answer to number 1, just to check. Number 2. Some of these are kind of long. Number 3. Number 4.

So let's just go through. We get-- in number 1, we get negative x e to the minus x minus e to the minus x plus c. Number 2, we get negative x squared over 2 times 1 plus x squared plus 1/2 natural log 1 plus x squared plus c. Three is x arc tan x minus 1/2 natural log of the quantity 1 plus x squared plus c, and four is negative natural log x over x minus 1 over x plus c.

So again, the whole point of this exercise, in my mind, is really to make sure we get a good understanding of, when we're doing integration by parts, which function makes the most sense to have as u, and which function makes the most sense to have as v prime. So that was the main point of this exercise. Hopefully you're starting to get a flavor for how these problems actually work. And I think I will stop there.