Introduction to the Cover-up Method

To integrate a rational function using the partial fractions method we must algebraically break it into parts. We're going to helped in this by a shortcut called the *cover-up method*.

We've disguised the easy integral:

$$\int \left(\frac{1}{x-1} + \frac{3}{x+2} \right) dx$$

as a harder one:

$$\int \frac{4x-1}{x^2+x-2} \, dx.$$

We'll use the method of partial fractions to unwind this disguise.

1. Our first step is to write down the integrand. Then we begin to undo the damage that we did by factoring the denominator (this can be a rather difficult step).
$$\frac{4x-1}{x^2+x-2} = \frac{4x-1}{(x-1)(x+2)}$$

2. Next we "set up" some unknowns, preparing to break the rational expression into pieces whose denominators are the factors we just found
$$\frac{4x-1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

3. The third step is to solve for the numerators; in this example they are A and B. Once we've completed this we've unwound the disguise.

The cover-up method makes step 3 more efficient and less clumsy.
We solve for A by multiplying by $x-1$.

$$\frac{4x-1}{x+2} = A + \frac{B}{x+2}(x-1)$$

Notice that we didn't try to clear the denominators completely; we just cleared one factor, which was all we needed to do to get A by itself.

If we plug in $x = 1$ we get:

$$\begin{aligned} \frac{4-1}{1+2} &= A+0 \\ \frac{3}{3} &= A \\ A &= 1 \end{aligned}$$

This is not a surprise — we saw at the start of the lecture that:

$$\frac{4x-1}{x^2+x-2} = \frac{1}{x-1} + \frac{3}{x+2}.$$

Question: Why did you choose $x = 1$?

Answer: Because it works really fast — that's part of the cover-up method. Notice that if we'd set $x = 1$ in the original equation we'd have gotten a denominator of zero; that wouldn't have helped us at all.

What we did was multiply both sides by $x - 1$ and then immediately set $x = 1$; that's like mutiplying both sides by zero! It turns out to be OK though because the equation is true *except* when $x = 1$, so what we're really getting is the limit as x approaches 1.

Setting $x = 1$ helps us by canceling out the term with the variable B in it.

We're going to learn a quicker way to do this in a second, but first let's find the value of B. To isolate B we multiply both sides by $x + 2$:

$$\frac{4x - 1}{(x - 1)(x + 2)} = \frac{A}{x - 1} + \frac{B}{x + 2}$$
$$\frac{4x - 1}{x - 1} = \frac{A}{x - 1}(x + 2) + B$$

Then we plug in $x = -2$:

$$\frac{4(-2) - 1}{(-2) - 1} = 0 + B$$
$$\frac{-9}{-3} = 0 + B$$
$$B = 3$$

We can now replace A and B by the values we've found to conclude that:

$$\frac{4x - 1}{x^2 + x - 2} = \frac{1}{x - 1} + \frac{3}{x + 2}.$$

What we've seen here is why the cover-up method works. Next we'll see how we can make it work faster.

Question: Can we use this method if the exponents on x aren't whole numbers? Will it work on square roots?

Answer: This only works with polynomials, which have whole numbered powers of x. For example, $x^2 + x - 2$ has powers of 2, 1 and 0 in the denominator. It won't work with square roots.

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