$\int \sin^n x \cos^m x\, dx, m = 1$

You already know something about integrating trigonometric functions; you can reverse anything you know about the derivative of a trigonometric function to get a fact about antiderivatives.

$$\begin{aligned} d\sin x = \cos x\, dx \quad &\Rightarrow \quad \int \cos x\, dx = \sin x + c \\ d\cos x = -\sin x\, dx \quad &\Rightarrow \quad \int \sin x\, dx = -\cos x + c \end{aligned}$$

Our plan is to use these two integration formulas and a few trig identities to derive more complicated formulas involving trig functions.

Our first topic is integrals of the form:

$$\int \sin^n x \cos^m x\, dx,$$

where m and n are non-negative integers. Integrals like this appear in Fourier series, among other places.

There are two cases to think about here. The easy case is the one in which at least one exponent is odd.

Example: $m = 1$

The trick in calculating $\int sin^n(x)\cos(x)\, dx$ is to make the substitution $u = \sin x$, so $du = \cos x\, dx$.

$$\begin{aligned} \int sin^n(x)\cos(x)\, dx &= \int u^n du \\ &= \frac{u^{n+1}}{n+1} + c \\ &= \frac{\sin^{n+1} x}{n+1} + c \end{aligned}$$

Although the answer $\frac{u^{n+1}}{n+1} + c$ looks nice, you need to reverse your substitution and plug in $\sin x$ for u at the end because while you may know what u is, your grader or employer does not.

What made this problem easy is that $\cos x$ is the derivative of $\sin x$.