Trapezoidal Rule

$$\text{Area} \approx \Delta x \left(\frac{y_o}{2} + y_1 + y_2 + ... + y_{n-1} + \frac{y_n}{2} \right).$$

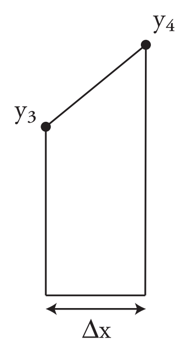

Figure 1: Approximation by areas of trapezoids.

The trapezoidal rule divides up the area under the graph into trapezoids (using segments of secant lines), rather than rectangles (using horizontal segments). As you can see from Figure 1, these diagonal lines come much closer to the curve than the tops of the rectangles used in the Riemann sum.

Remember that the area of a trapezoid is the area of the base times its average height. When applying the trapezoidal rule, the base of a trapezoid has length Δx and its sides have heights y_{i-1} and y_i; trapezoid i has area $\Delta x \frac{y_{i-1}+y+i}{2}$.

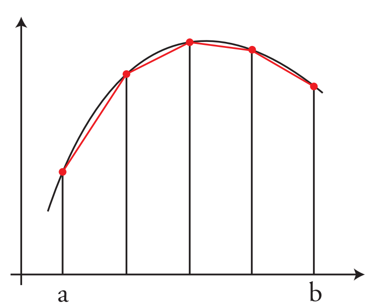

Figure 2: Area $= \left(\frac{y_3+y_4}{2}\right)\Delta x$.

When we add up the areas of all the trapezoids under the curve, we get:

$$\text{Area} \quad = \quad \Delta x \left\{ \frac{y_o + y_1}{2} + \frac{y_1 + y_2}{2} + \frac{y_2 + y_3}{2} + ... + \frac{y_{n-1} + y_n}{2} \right\}$$

$$= \quad \Delta x \left(\frac{y_o}{2} + y_1 + y_2 + ... + y_{n-1} + \frac{y_n}{2} \right).$$

Notice that the trapezoidal rule is the average of the left Riemann sum and the right Riemann sum; it gives a more symmetric treatment of the endpoints a and b than a Riemann sum does.

This looks good and in fact it is much better than a Riemann sum; however, it's still not very efficient.