Comparison With Approximation

We just used l'Hôpital's rule to show that:

$$\lim_{x \to 0} \frac{\cos x - 1}{x^2} = -\frac{1}{2}.$$

Let's compare that to what we get using the method of approximations, replacing the functions involved with their linear or quadratic approximations.

In the example of $\lim_{x \to 0} \frac{\sin 5x}{\sin 2x}$, we would use the linear approximation $\sin u \approx u$ for u near 0 to get:

$$\frac{\sin 5x}{\sin 2x} \approx \frac{5x}{2x} = \frac{5}{2}$$

for x near 0. Since the approximation $\sin u \approx u$ becomes exact as u approaches 0, we could then conclude that:

$$\lim_{x \to 0} \frac{\sin 5x}{\sin 2x} = \frac{5}{2}.$$

To use this method to find:

$$\lim_{x \to 0} \frac{\cos x - 1}{x^2},$$

we would approximate:

$$\frac{\cos x - 1}{x^2}$$

by:

$$\frac{(1 - x^2/2) - 1}{x^2} = \frac{-x^2/2}{x^2} = -\frac{1}{2}.$$

Again the approximation becomes exact as x approaches 0, so:

$$\lim_{x \to 0} \frac{\cos x - 1}{x^2} = -\frac{1}{2}.$$

The method of approximation gives us the same result as l'Hôpital's rule, as it should. Both methods are valid and they both involve about the same amount of work. The approximation we used for the cosine function is related to the second derivative of $\cos x$, and we had to find the second derivative of $\cos x$ when we applied l'Hôpital's rule.

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