

PROFESSOR: Welcome back to recitation.

In this segment I'm going to actually show-- well, you're actually going to show-- the derivative of tangent x using the quotient rule. So what I'd like you to do, I wanted to remind you of what the quotient rule is. So u and v are functions of x . We want to take the derivative of u divided by v . I've written the formula that you were given in class for this. And I'm asking for you to take d/dx of tangent x using the quotient rule. And the hint I will give you is the reason we can obviously use the quotient rule is because tangent x is equal to a quotient of two functions of x . It's sine x divided by cosine of x .

So I'm going to give you a minute to work this out for yourself, and then when we come back I will do it for you.

OK. So we want to find the derivative of tangent x . So let me-- let me work on this side of the board. So I'm actually going to take d/dx of sine x divided by cosine of x .

OK. So in this case, sine x is u . Cosine x is v . So using my quotient rule I know that first I have to take the derivative of sine x -- that's cosine x -- and then I multiply it by the denominator, the v , which is cosine x . So my first term in the numerator is cosine squared x . Again, one cosine x comes from the derivative of sine x , one cosine x is the v . It's the cosine x in the denominator.

Then I have to subtract v prime u . The derivative of cosine x is negative sine x . I'll actually just write that one down. And then I bring the u along for the ride. So I multiply by sine x here. And then I take v squared in the denominator from the formula. v , again, is cosine x , so I take cosine squared x in the denominator.

Now this at this point is a little bit messy, but the nice thing is that we can use some trigonometric identities to simplify this. So let me first write out what it is a little more clearly. Minus a negative gives you a positive. And then here I get sine x times sine x , so I get the sine squared x . And then I keep divided by cosine squared x .

Now at this point some of you might have divided by cosine squared x here and gotten 1, and divided by cosine squared x here and gotten tangent squared x . And then from there you could simplify to another trigonometric function. I'm going to go straight a different way to show you what that actually also equals. So there, at this point I want to stress there are sort of two ways you can get to the same place.

But I'm going to use the fact that the numerator is a very nice trigonometric identity that we know. We know cosine squared x plus sine squared x always equals 1. So this is quite lovely, the numerator simplifies to 1, the denominator stays cosine squared x . What is this function?

1 over cosine x is actually secant x . So if you need, at this point, to rewrite the whole thing like this, right? 1 squared is 1 and in the denominator we still get cosine squared x -- this tells you that 1 over cosine squared x is actually just equal to secant squared x . So again, what I want to point out is we've now taken the derivative of tangent x and we got that that's secant squared x .

Now using this quotient rule, you can do the same kind of thing with cotangent x , with cosecant x , with secant x . You can find all these derivatives of these trigonometric functions using the quotient rule. So if you want to know what the derivative of secant x is, you should take $d(dx$ of 1 divided by cosine x and use the quotient rule. Or, in fact, the chain rule would work well there also, to find that derivative.

So we are building up the number of derivatives we can find using these different rules.

So we'll stop there.