Derivative of $\arcsin(x)$

For a final example, we quickly find the derivative of $y = \sin^{-1} x = \arcsin x$.

As usual, we simplify the equation by taking the sine of both sides:

$$\begin{aligned} y &= \sin^{-1} x \\ \sin y &= x \end{aligned}$$

We next take the derivative of both sides of the equation and solve for $y' = \frac{dy}{dx}$.

$$\begin{aligned} \sin y &= x \\ (\cos y) \cdot y' &= 1 \\ y' &= \frac{1}{\cos y} \end{aligned}$$

We want to rewrite this in terms of $x = \sin y$. Luckily there is a simple trig. identity relating $\cos y$ to $\sin y$. We can solve it for $\cos y$ and "plug in".

$$\begin{aligned} \cos^2 y + \sin^2 y &= 1 \\ \cos^2 y &= 1 - \sin^2 y \\ \cos y &= \sqrt{1 - \sin^2 y} \quad (\cos y > 0 \text{ on the range of } y = \sin^{-1} x) \end{aligned}$$

Plugging this in to our equation for $y' = \frac{d}{dx} \sin^{-1} x$ we get:

$$y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1 - \sin^2 y}} = \frac{1}{\sqrt{1 - x^2}}.$$

Notice that we made a choice between a positive and negative square root when solving for $\cos y$. We chose the positive square root because we usually define $\sin^{-1} x$ to have outputs between $-\pi/2$ and $\pi/2$, and the cosine function is always positive on this interval.

When dealing with inverse functions we are often faced with choices like this; when in doubt draw a graph and be sure your choices make sense in the context of your problem.