PROFESSOR: Welcome back to recitation. In this video, what I'd like us to do is, do a little bit of practice with sigma notation. So this will be just a few short problems to make sure that you're comfortable with what all the pieces in the sigma notation actually do. We're going to start with two problems here.

And the first one is going to be a fill-in-the-blanks type of problem. And the object is, I've given you a sum on the left-hand side, and then I've given you two other sums, but I've left in each place two blanks, and I've filled in the rest. You have enough information to fill in the two blanks. So what I'd like you to do in this problem is fill in the two blanks so that the sums are equal. And the object is obviously is to do this without writing out all the terms and adding up and then going backwards. So you really want to try and understand what each part of the sigma notation does.

The second problem I'd like you to do is a simplification problem. There are three finite sums. And what I'd like you to do is combine them into a single sum or two sums. Do the best you can to get it as simplified as you can without actually writing out a number, but keeping it in some sort of notation form. So the object is just to combine what you can and simplify where you can.

And then we'll do another one in a little bit. But first let's do these two. I'll give you a while to work on them and then I'll be back.

OK, welcome back. We're going to start with the first problem. So the idea is to really understand what each of these pieces represents. And let's look at the first sum and make sure we understand what's going on. So we have 2 raised to a power. And what we do is we index over k from 1 to 5. So we're going to take 2 to the first, plus 2 to the second, all the way up to 2 to the fifth. And that's where the sum stops.

Now in this summation, k is indexed from some number I haven't told you yet, up to 7. And I didn't specify what power of k we want. So there are a couple ways you can think about this. It's maybe easiest to work from what we have up here. We know that the exponent, last exponent we would like on the power of 2 is a 5 in the end. But right now, if we just put a k here, the power would be 7.

So what we'd like to do is make whatever the power is up here-- based on that 7-- we'd like that power to be 2 less than the number that we're putting in there. So probably we would like this to be a k minus 2, because notice then, the last number you put in, you get a 5. Which corresponds to the last number you put in here, a 2 to the fifth. Now the last number here is 2 to the fifth.

And this now will dictate what we put in the blank down here. Because the first value of k we wanted here, the first term we wanted in this sum, was 2 to the first. So the first term we want in this sum is going to be 2 to the first. So that means that we would like k to start at 3.

Another way to think about this is that we know we want the same number of values that we're summing over. So notice that from 1 up to 5, there are 5 values we're summing over. From 3 up to 7, there are actually 5 values we're summing over. You might think there are 4, because 7 minus 3 is 4, but you actually have to count: 3, 4, 5, 6, 7. You see in fact there are 5 values there. So don't get confused by that. The differences are the same. 5 minus 1 is 4. 7 minus 3 is 4. So that's good. We have the same number of things we're summing up over. And the first terms are the same. And then you notice, because of the way we've written, it actually is going to be exactly equal. You could expand and check. but these are going to be equal sums.

Now the third one, I was a little trickier, maybe. I pulled out a factor of 2. And so now what we've done is we've taken one of the 2's that was in all of those terms and we pulled it out. Right? So what do we have here? Well we still have 2 to the k. But what does this actually equal? To make it easier on myself, I'm going to rewrite this in another way. If I pull the 2 back in, I get a 2 to the k plus 1.

So now what I've done is I've given you this 2 pulled out. What it's actually doing is it's changing the exponent value. But again, what do we want the exponents to run over? We want them to start, this exponent to start at 1 and to end at 5. So to get it to start at 1 and end at 5, I need k to be 0 to start, and finish at 4. And that will be sufficient.

Now again, let's just make sure that this makes sense to us. If k is 0, I get 2 to the 0 here. But when I multiply by a 2 in front, the first term is 2 to the first. Which is the first term here. Let's just check one more to make sure we feel good about it. When k equals 1, I get a 2 to the first here, times 2. So that's a 2 squared. That's the second term in this sum is 2 squared. The second term in this sum is when I put in k equals 2, I get a 2 squared.

So we see in fact that I've chosen these values in blue. Now these three sums are actually equal. If you're still nervous about it, maybe you can expand the sums and look at them and notice that they are indeed going to work.

Now what I'd like us to do is work on simplifying a problem. And if you'll notice, I've put in three sums, the values here, two of them are from 1 to 100, one of them is from 45 to 100. And the three different things that I'm summing: n cubed minus n squared, n cubed minus n squared minus n, and then n. And I wanted us to simplify this as much as we could.

Now because these are finite sums we can split up over the terms, as long as we keep the right index.

So let me actually use the regular chalk for this, and I'm going to look at how I can split up the second term to help with the first and the third. So in the second term, notice I have an n squared and an n cubed-- or n cubed minus n squared here, and an n cubed minus n squared here. So what I can do is, I'm going to look at those terms together. And then I'm going to look at the n, the terms-- or summation with n and the summation with n. And we'll compare them.

So let me write out what we get. We're going to leave the first one alone for the moment. And then I'm going to subtract off this part of that summation. And what's left in that summation is every term I had a minus n also. So I'm going to pull that minus out with this negative. And what I'm doing is I'm taking 45-- n equals 45 to 100 of these added up. And then n equals 45 to 100 of this added up.

So I end up with another term. n equals 45 to 100 of just n. So those two terms are coming from the middle one split into two pieces. And then the last term, I just write down.

So now, it's set up to go nicely for this into a single summation. And this into a single summation. And then we'll see if we can combine them further. So if you look here, I have n equal 1 to a 100 of a sum. And then I have n equal 45 to a 100 of the same sum. What's that actually mean? That means I'm seeing the 45 to 100 thing here, and here. And there's a difference. Right? So I plug in n equals 45 here. I get 45 to the third minus 45 squared. I plug in n equals 45 here, I get the same thing. And I'm subtracting.

So what's actually happening is all the terms that have, that show up in both this sum and this sum are being subtracted off. What are those terms? Those are all the terms for n equal 45 up to 100. Because it's in this summation and this one goes all the way from 1 to a 100. So it certainly includes 45 to 100.

So, in fact, you see that all you wind up with in the end is n equals 1 to 44 of n cubed minus n squared. Again why is that? This has the 1 through 44 terms and it has the 45 to 100 terms. This has the 45 through 100 terms only. So the 45 to 100 terms are in both and they're subtracted off. So that's one way to think about why we end up with n equals 1 to 44 of this sum.

And then let's look what we get here. Well in fact, we see it's exactly the same kind of thing. This is 45 to 100. This is 1 to 100. But notice now that the minus is on the 1 to 100 part. So I'm actually going to get negative of n equals 1 to 44 of n. Because the 45 terms here, 45 to 100, are in both. So the 45 to 100 here, subtract off the 45 to 100 here. Those all go away. But I'm still left with the minus n equals 1 to 44.

And now I could simplify this further, if I wanted, into a single sum. 1 to 44 n cubed minus n squared minus n. Why can I do that so easily? They're indexing over the same values. That's an important point. If this was indexing over different values, I'd have to change this formula in order to substitute it in. But because they're indexing over exactly the same values, I can just take these two pieces and put them into a single sum.

So we're going to stop those two problems now. We're going to do one more summation notation problem. So we're going to come over here. And I'm just going to ask you to write, this is a sum of five terms. I'm going to ask you to write this in sigma notation. And the main thing-- there will be multiple ways to do this. So you might come up with a different answer than I do. But I'd like you to work on it for a few minutes. And then when you feel confident, come back and I will show you how I solved the problem.

OK, welcome back one more time. We're going to try and put this in sigma notation. And I have to tell you that when I look at this kind of problem, and I see the same kind of factor in each of these things, I like to make it as simple on myself as possible. I like to pull out that factor just to make sure that I can simplify this as much as possible before I go into sigma notation.

So the common factor to all of these is 1/5. I'm going to pull out a 1/5 before I start doing anything else. There I get a 1. There I get a minus 1/2 plus 1/3 minus 1/4 plus 1/5. Now, if you couldn't do it before, you can probably do it now. Because now it's sort of very obvious how these terms are changing. So we want to see how these terms are changing and how we could index them in some variable.

So let's start with the 1/5 and I'll start with my summation and then we'll figure out what all the pieces are. Now obviously the numerator in this case is fixed at 1-- and I've got a fraction here, so the numerator's fixed at 1-- but the sign is alternating. So how do you alternate sign? You're going to take negative 1 and raise it to a power. Now the power you raise it to will depend on if you want the first term to be positive or negative, and where you start your summation. So there's a lot of choices you can make. But I'm going to start my summation, we'll say, we'll do it in k and we'll start at k equals 1. And then we'll have to figure everything out from that. So I'm going to start my summation at k equals 1. My first term, I want to be positive 1. So I need my power to be k plus 1. Because now my power here is going to be-- when I put in a 1, I get a 1 plus 1, I get 2. Negative one squared is positive. That's that's what I want.

You might have done k minus 1. If you did k minus 1, that's OK. Because k minus 1 is also an even number. So when I take negative 1 and I square it, I still get a positive number. So there are a lot of choices one can make and still be correct on that power. And then, I'm counting up. Notice the denominator is increasing just by 1 each time. And so it looks like, I could do just something like over k.

Now let's check if that makes sense. Well when k is 1, I get 1 in the denominator. This is 1 over 1. When k is 2, I get 2 in the denominator. When k is 3, I get 3 in the denominator. So that looks good. And now the only question is, where should I stop this thing? So I have my alternating sign. My denominator looks right. For what value of k do I want to stop? I want to stop when the denominator equals 5. And so I just need to put a 5 up here. And then I'm done.

Now, if you wanted to move the 1/5 back in, you could actually do that. Maybe your solution looked something-- I pull the 1/5 back in. And I have 5k in there instead. Maybe that was your solution. But these are ultimately the same thing. Because really this is just distributing. Right? This is a big sum. I have a 1/5 out in front. And so I multiply every term by 1/5. So I just have to put a 5 in the denominator. So you might have had something more like this. That's still correct. So just to stress, that really the sigma notation, it's a good tool to understand how to manipulate easily. So there are probably more problems you can find to practice, if you're nervous about this. But I just wanted to give you a chance to see a couple of them and how we work on them. That's where I'll stop.