CHRISTINE BREINER: Welcome back to recitation. In this segment, we're going to talk about the product rule for three functions and then we're going to do an example. And what I want to do first is remind you the product rule for two functions, because we're going to use that to figure out the product rule for three functions.

So throughout this segment, we are going to assume that u and v and w are all functions of x. So I'm going to drop the of x just so it's a little easier to write. This notation should be familiar with things you saw in the lecture. So, for two functions, let me remind you.

If u*v, the product, and you take its derivative, so prime will denote d/dx. Then we can take the derivative of the first times the second function left alone, plus the derivative of the second function times the first left alone. So this should again be familiar from class. And now what we want to do is expand that to the product of three functions, u times v times w. And we're going to explicitly use this rule. So, u*v*w prime is what we want to look at.

So we're just going to take advantage of what we know to figure out what this expression will be. What this product of three functions when I take its derivative will be. So in order to do this easily, what we're going to do is treat v times w as a single function. OK? So v times w will be our second function that essentially takes the place of the V up here.

So using the product rule for two functions, what I get when I take this derivative, is I get u prime times v*w plus, I take the derivative of this second thing, which is v*w prime. And then I leave u alone. OK?

We're not quite done, but you can see now, again if we compare to what's above, you take the derivative of the first function, you leave the second function alone. You take the derivative of the second function, you leave the first function alone. But now again, what do we do here? Well we have a product rule for two functions, so let's use it.

So, I'll leave the first thing alone, u prime-- oops, that does not look like a v-- v*w plus, now let's expand this. Take the derivative of the first function there. That's v prime. I leave the w alone. Plus the derivative of the second function. That's w prime. I leave the v alone. And I keep the u there. OK?

I'm going to just expand and write it in a nice order, so we can see sort of exactly what happens. So, u prime v*w plus v prime u*w plus w prime u*v. So what you can see here is, what happens? You take the derivative of the first function, you leave the second and third alone. Then you take the derivative of the second function, you leave the first and third alone. Then you take the derivative of the third function, you leave the first and second alone. And you add up those three terms.

I would imagine that at this point you anticipate a pattern. So if I had a fourth function. If I did u times v times w times z, let's say. And I took that derivative with respect to x. You could probably anticipate, you would have four terms when you added them up. And that fourth term would have to include a derivative of the fourth function.

So from here, actually you can probably tell me what the derivative of the product of n functions is. And you can check it using the same kind of rule. But what we're going to do at this point, is we're going to just make sure we understand this. We're going to compute an example. So since we know products-- or we know derivatives of powers of x, and we know derivatives of the basic trig functions, we'll do a product rule using those functions.

So, let me take an example. So we'll say, f of x equals x squared sine x cosine x. OK? And I want you to find f prime of x. OK, I'm going to give you a moment to do it. You should probably pause the video here, make sure you can do it, and then you can, you can restart the video when you want to check your answer.

OK, so we have a product rule for three functions, we have an example that I asked you to determine and gave you a moment to do it. So now I will actually work out the example over here to the right. So I will determine f prime of x.

Now what are our three functions? Well we have x squared is the first, sine x is the second, cosine x is the third. So we'll have three terms. The first term has to have the derivative of the x squared. That's going to give me a 2x. And I leave the other two terms alone. So I have 2x sine x cosine x plus-- I may want to just write these below. OK.

Now in the next term, I should take the derivative of the sine x. And leave the x squared and the cosine x alone. The derivative of sine x is cosine x. So I'm actually going to write this underneath. So we'll have-- I'm going to put the plus underneath also so we remember it's a sum. Plus, so the derivative of sine x is cosine x. And then we have a times x squared times-- oops-- another cosine x, the third function. OK?

And then the third term, I take the derivative of the third function and I leave the first and second alone. The derivative of cosine x is negative sine x. So I actually have a negative sine x times x squared times the sine x here.

I can do some simplifying if I want. But maybe, if I were trying to write this nicely for someone who was reading mathematics, I would put all of the polynomials in front and all of the coefficients in front. So to be very kind to someone, I might write it like this. And notice cosine x cosine x is cosine squared x. And then minus x squared sine squared x.

And there are other ways, I could rewrite this and using trig identities. But this is a sufficient answer at this point. So this is actually a good way to write the derivative of that function, f of x. And this is where we'll stop.