Working with exponents

We start out with "base" number a. This number a must be positive, and we're going to assume $a > 1$ to make it easier to draw graphs.

What is the derivative of a^x? We'll start to answer this question by reviewing what we know about exponents.

To begin with, we know that:

$$a^0 = 1; \quad a^1 = a; \quad a^2 = a \cdot a; \quad a^3 = a \cdot a \cdot a \quad \ldots$$

In general,

$$a^{x_1 + x_2} = a^{x_1} a^{x_2}$$

Together with the first two properties, this describes the exponential function a^x.

From these properties, we can derive:

$$(a^{x_1})^{x_2} = a^{x_1 x_2}$$

and we can easily evaluate a^n for any positive integer n. For negative integers, we can see from the fact that $a^m \cdot a^{-m} = a^{m-m} = 1$ that $a^{-m} = \frac{1}{a^m}$.

We want to be able to evaluate a^x for any number x; not just for integers. We start by defining a^x for rational values of x:

$$a^{\frac{p}{q}} = \sqrt[q]{a^p} \quad \text{(where } p \text{ and } q \text{ are integers.)}$$

Since $a^{1/2} \cdot a^{1/2} = a^1 = \sqrt{a} \cdot \sqrt{a}$, this seems like a reasonable definition.

All that's left is to define a^x for irrational numbers; we do this by "filling in" the gaps in the function to make it continuous. This is what your calculator does when you ask it for the value of $3^{\sqrt{2}}$ or 2^{π}. It can't give you an exact answer, so it gives you a decimal (rational) number very close to the exact answer.

Take some time and sketch the graph of 2^x to "get a feel" for how exponential functions work.