

Volume of a Wine Glass: Vertical Slices

If we use vertical slices to compute the volume of our exponential wine glass, we'll be adding up volumes of shells with height $e - y$, radius x and thickness dx .

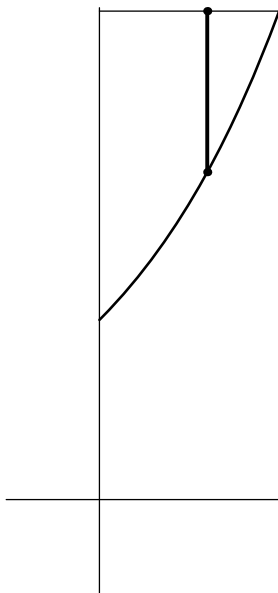


Figure 1: Rotating a slice of thickness dx about the y -axis produces a shell.

$$\begin{aligned}
 \text{Volume} &= \int_0^1 (e - y)2\pi x \, dx \\
 &= \int_0^1 (e - e^x)2\pi x \, dx \\
 &= \int_0^1 2\pi e x \, dx - \int_0^1 2\pi x e^x \, dx \\
 &= \underbrace{\frac{2\pi e}{2}}_{\text{area of a triangle}} - 2\pi G_1(x) \Big|_0^1 \\
 &= \pi e - 2\pi [x e^x - e^x]_0^1 \\
 &= \pi e - 2\pi [(e - e) - (0 - 1)] \\
 &= \pi e - 2\pi \\
 &= \pi(e - 2).
 \end{aligned}$$

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