

**Integrating**  $\frac{1}{(5x+2)^2}$  **from 1 to infinity**

Compute:  $\int_1^\infty \frac{dx}{(5x+2)^2}$

**Solution**

This is an improper integral; our first step is to convert it to a proper one:

$$\int_1^\infty \frac{dx}{(5x+2)^2} = \lim_{N \rightarrow \infty} \int_1^N \frac{dx}{(5x+2)^2}$$

We can now apply a substitution to evaluate the integral. If we let  $u = 5x+2$  then  $du = 5 dx$  and  $dx = \frac{1}{5} du$ . When  $x = 1$ ,  $u = 7$  and when  $x = N$ ,  $u = 5N+2$ . Therefore:

$$\begin{aligned} \int_1^N \frac{dx}{(5x+2)^2} &= \int_7^{5N+2} \frac{\frac{1}{5} du}{u^2} \\ &= \frac{1}{5} \int_7^{5N+2} u^{-2} du \\ &= \frac{1}{5} (-u^{-1}) \Big|_7^{5N+2} \\ &= \frac{1}{5} \left( -\frac{1}{5N+2} - \left( -\frac{1}{7} \right) \right) \\ &= \frac{1}{35} - \frac{1}{5(5N+2)} \end{aligned}$$

By evaluating the limit, we find the value of the improper integral:

$$\begin{aligned} \int_1^\infty \frac{dx}{(5x+2)^2} &= \lim_{N \rightarrow \infty} \int_1^N \frac{dx}{(5x+2)^2} \\ &= \lim_{N \rightarrow \infty} \left( \frac{1}{35} - \frac{1}{5(5N+2)} \right) \\ &= \frac{1}{35} - 0 \\ &= \frac{1}{35} \approx .03 \end{aligned}$$

Geometrically, this tells us that if we horizontally compress the graph of  $\frac{1}{x^2}$  (by multiplying  $x$  by 5) and then shift the result to the left 2 units, the final graph is very close to the  $x$ -axis for  $x > 1$ .

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18.01SC Single Variable Calculus  
Fall 2010

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