Review of Riemann Sums

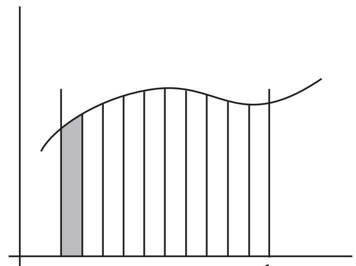

Figure 1: The area under the curve is divided into n regions of equal width.

As was mentioned at the start of this unit, Riemann sums approximate the area between the x-axis and a curve over the interval $[a, b]$ by a sum of areas of rectangles. Each rectangle has width $x_i - x_{i-1} = \Delta x$; there are n rectangles whose sides have x-coordinates $a = x_o < x_1 < x_2 ... < x_n = b$. The heights of the rectangles are $y_o = f(x_o),\ y_1 = f(x_1), ..., y_{n-1} = f(x_{n-1})$ (if the left edge of each rectangle is exactly as high as the graph).

Our goal is to "average" or add these y-values to get an approximation to

$$\int_a^b f(x)\,dx.$$

The formula for the (left) Riemann sum is:

$$(y_0 + y_1 + \ldots + y_{n-1})\Delta x.$$

If we let the right hand side of each rectangle be as high as the graph, using right endpoints instead of the left endpoints, we get the right Riemann sum:

$$(y_1 + y_2 + \ldots + y_n)\Delta x.$$