# 16.410-13 Problems Set 7
Due in class on Wed. Nov. 10th, 2010.

Problem 1: Model-based Diagnosis (20 points)

Part A

Consider a Mars rover on which the diagnosis algorithm that you have learned in the class is running.

The electrical system components of the rover are shown in the figure below. There are 3 power sources (shown as P1, P2, and P3), two regulators (R1 and R2), and two sensors (S1 and S2) of the electrical system (the two wheels are not a part of the electrical system).
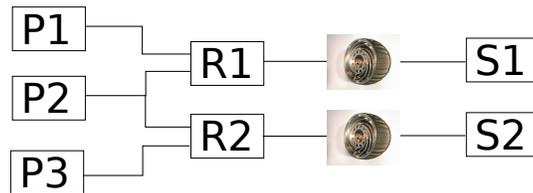
Mars rover wheel image courtesy of E. V. Day. Used with permission.

In normal operation, the power sources provide power whenever they are on. A regulator provides the wheel commands whenever *both* power sources it is connected to is providing power. The wheels move when they are commanded by the regulator. The sensors read "movement detected" when the wheels move; they read "no movement" when the wheels are not moving.

If a power source is faulty, it may or may not provide power when turned on. If a regulator is faulty, it may not provide the wheel commands, even though both power sources it is connected to are providing the power; or, it may provide wheel commands even though one (or both) of the power sources is not providing power. The sensors may fail and provide faulty outputs (different than the actual condition of the wheels).

The wheels move if they are commanded. They are very good wheels, and are never faulty.

Part A.1

All the power sources are on, however your sensor readings are

- S1: no movement

- S2: movement detected

Show how your diagnosis algorithm works on this problem. Identify all the conflicts, find the constituent kernels, and compute the minimal kernel using the set covering algorithm.

Part A.2

Assume that, in its normal operation mode, a regulator provides the wheel commands if *at least one of the power sources (not necessarily both)* are providing power. For the same sensor readings (given in Part A.1). Show how your diagnosis algorithm works. Identify all the conflicts, find the constituent kernels, and compute the minimal kernel.

Please indicate the time it took for you to solve each part of the problem.

Problem 2: Optimal CSPs and Conflict directed A* (20 points)

Part A

Recall that a constraint satisfaction problem (CSP) can be defined as follows

Definition 1 *A CSP is defined by a triple $\langle X, D, C \rangle$, where*

- $X = \{x_1, x_2, \ldots, x_n\}$ *is a set of variables,*

- $D = \{D_1, D_2, \ldots, D_n\}$ *is a set of domains for each variable, i.e., D_i is the domain of variable x_i,*

- $C = \{R_{i,j} \,|\, i, j \in \{1, 2, \ldots, n\}, \ i \neq j\}$ *is a set of binary constraints.*

The constraint satisfaction problem is to find a value $d_i \in D_i$ for variable x_i for each $i \in \{1, 2, \ldots, n\}$, such that all the constraints are satisfied, i.e., $(d_i, d_j) \in R_{i,j}$ for all $i, j \in \{1, 2, \ldots, n\}$ with $i \neq j$.

Augment the definition of CSP to the optimal CSP. Provide the optimal CSP description in a formal format as shown above. You should add in a cost function (you can denote that by g), and provide the definition of variables, domain, and constraints as shown above. Consider only binary constraints.
(HINT: Just augment the above definition with the cost function, and modify the "task" of the problem).

Part B

The optimal CSP framework is in fact very powerful for solving several combinatorial optimization problems. Once encoded in the form of an optimal CSP, the problem can be solved using, e.g., a variant of conflict-directed A* search. In this part, you will encode a combinatorial optimization problem as an optimal CSP.

Consider a version of the optimal CSP problem in which constraints with arbitrary arity are allowed. That is, the constraints do not have to be binary, but you can use constraints that involve two or more variables. Such a constraint, e.g., one that binds three variables, can be defined as a relation R that is a set of triples, i.e., $R = \{(x_1, y_1, z_1), (x_2, y_2, z_2,), \ldots, (x_k, y_k, z_k)\}$, where x_i, y_i, z_i are the allowed values for the variables x, y, and z.

Here is our problem. We have finally set up our moon post that includes many sites (for those who are taking 16.413, later on we will use our activity planners to work on the logistics problem of carrying materials between these sites). Next, we would like to build s connecting the sites. We would like to do this in such a way that all the sites are connected, while spending minimal amount of money. A figure showing all the sites on the moon and potential roads that we can construct is given in the next page. The price for constructing these roads are also given in the figure (in billions of dollars).

A related problem, and an important problem in combinatorial optimization, is called the *minimum spanning tree*. The problem can be defined as follows. Recall that a graph $G = (V, E)$ is defined by a set of vertices V and a set of edges E. In this problem we are only interested in undirected graphs. A tree is a special type of graph which includes no (undirected) cycles. An spanning tree is one that connects all the nodes. A minimum spanning tree is a spanning tree such that the sum of the cost of all the edges is minimized. The figure below illustrates a minimum spanning tree.

In the figure, the graph edges in the original graph are shown in gray. The edges of the minimum spanning tree are shown in black. Notice that every pair of nodes is connected with a path through the minimum spanning tree. Moreover, the cost of this tree is minimal when compared to any other subgraph that includes all the vertices.
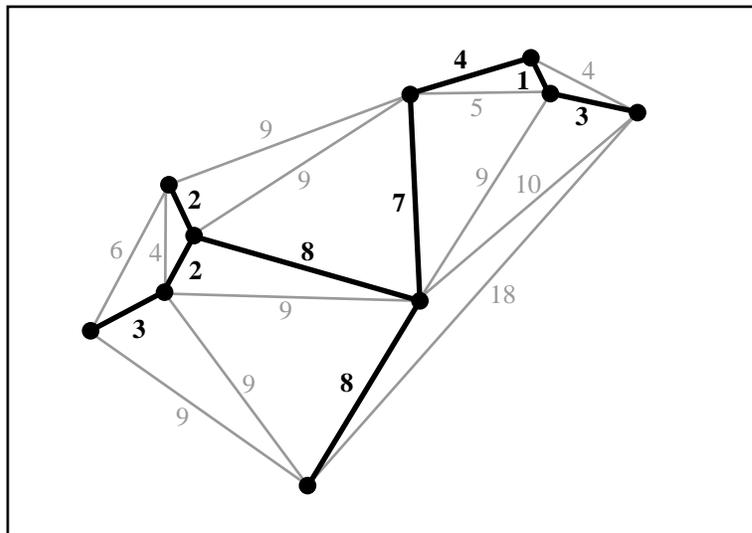
Image by MIT OpenCourseWare.

As an expert in optimal CSPs, the Moon Initiative Co. comes to you asking for a solution to this problem. Please provide an encoding of this problem as an optimal CSP. Do not try to solve the problem (The Moon Initiative Co. has many engineers to its disposal to run it with a conflict-directed A* – they are just missing the expert who knows the encoding).
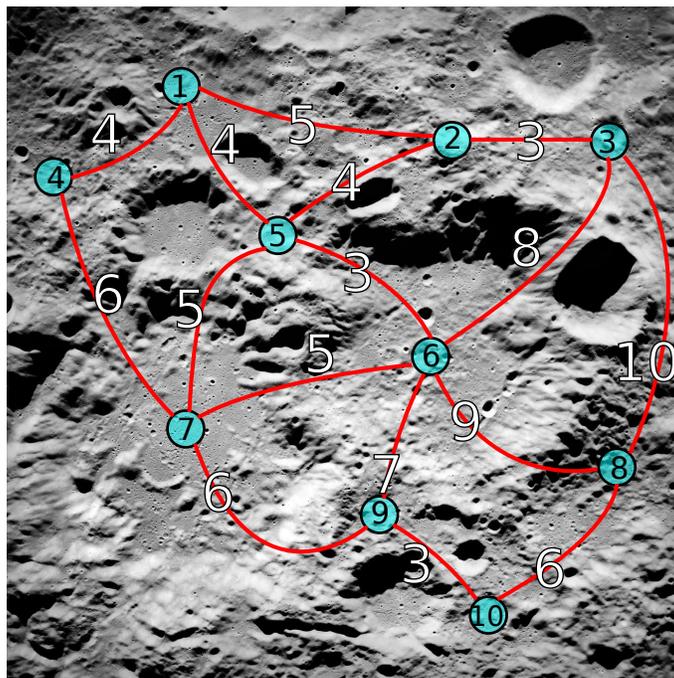
Part C: Conflict-directed A*

Please explain each of the following in one paragraph.

Probabilistic diagnosis: Briefly explain the motivation behind probabilistic diagnosis.

Kernels: Briefly explain how best kernels are generated in the conflict-directed A*.
    Please indicate the time it took for you to solve each part of the problem.

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