S3

Determine + plot $C_L(t)$

$$C_L(t) = 2\pi\alpha_0\,\psi(\bar{t}) \qquad \bar{t} = \frac{2Ut}{c}$$

$$\psi(\bar{t}) = \begin{cases} 0, & \bar{t}<0 \\ 1-\frac{1}{2}e^{-0.13\bar{t}} - \frac{1}{2}e^{-\bar{t}}, & \bar{t}\geq 0 \end{cases}$$

Flying at nominal $\alpha=0$ w/ small deflections

$$\alpha = \frac{w}{U}$$

Using known values $c = 1\,m$, $U = 1\,m/s$

$$\alpha = w, \quad \bar{t} = \frac{2Ut}{c} = 2t$$

So $w(t)$ is our input function

$$w(t) = \begin{cases} 0 & , t<0 \\ 0.1(1-e^{-2t}) & , t\geq 0 \end{cases}$$

And we can express $\psi(\bar{t})$ as $\psi(t)$

$$\psi(t) = \begin{cases} 0 & t<0 \\ 1-\frac{1}{2}e^{-0.26t} - \frac{1}{2}e^{-2t} & , t\geq 0 \end{cases} \qquad \bar{t}=0 \text{ when } t=0$$

We know that ψ is the step response so we can use Duhamel's Superposition Integral

$$y(t) = \psi(t)w(0) + \int_0^\infty \psi(t-\tau)\,w'(\tau)\,d\tau$$

$$w'(t) = \begin{cases} 0 & , t<0 \\ 0.2e^{-2t} & , t\geq 0 \end{cases} = 0.2e^{-2t}\sigma(t)$$

- non zero for all $t\geq 0$

$\psi(t-\tau)$ is only non-zero from $0\leq\tau\leq t$

So bounds of integration are from 0 to t

Now have

$$y(t) = Y(t)\,w(0) + \int_0^t Y(t-\tau)\, w'(\tau)\, d\tau$$

$$w(0) = 0.1\left(1 - e^{-2\cdot 0}\right) = 0$$

$$y(t) = \int_0^t \left(1 - \tfrac{1}{2}e^{-0.26(t-\tau)} - \tfrac{1}{2}e^{-2(t-\tau)}\right)\left(0.2e^{-2\tau}\right) d\tau$$

multiplying out:

$$y(t) = \int_0^t \left[0.2e^{-2\tau} - 0.1e^{-2\tau}e^{-0.26t}e^{0.26\tau} - 0.1e^{-2\tau}e^{-2t}e^{2\tau}\right] d\tau$$

$$= \int_0^t \left[0.2e^{-2\tau} - 0.1e^{-1.74\tau}e^{-0.26t} - 0.1e^{-2t}\right] d\tau$$

We can now integrate, noting that e^{t} terms are constant WRT τ

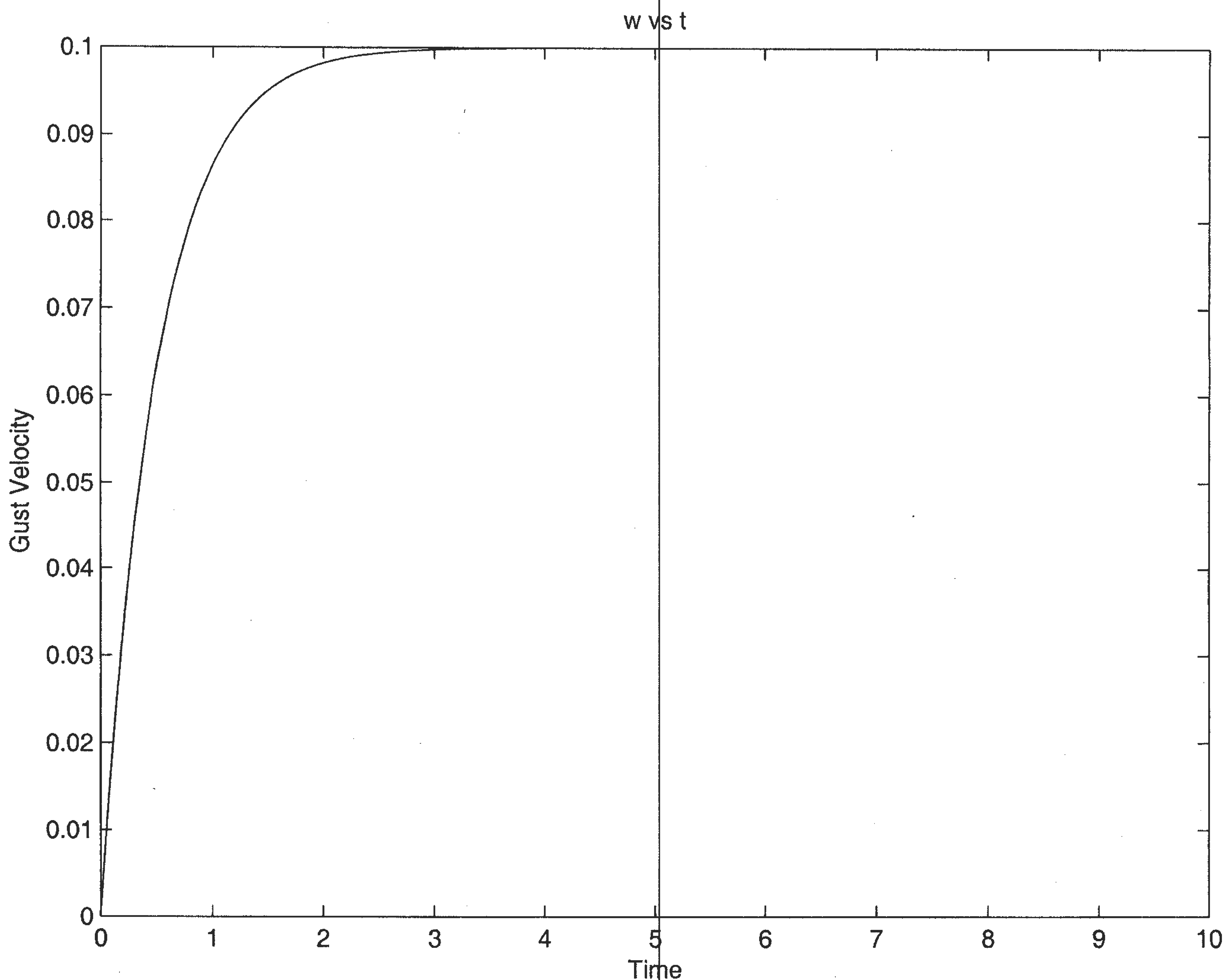
$$y(t) = \left(-0.1e^{-2\tau} + \frac{0.1}{1.74}e^{-1.74\tau}e^{-0.26t} - 0.1\tau e^{-2t}\right)\Bigg|_0^t$$

$$y(t) = -0.1e^{-2t} + \frac{0.1}{1.74}e^{-2t} - 0.1te^{-2t} + 0.1 - \frac{0.1}{1.74}e^{-0.26t}$$

$$y(t) = \left(-0.0425\,e^{-2t} - 0.0575e^{-.26t} - 0.1e^{-2t}t + .1\right)\sigma(t)$$

and $C_L = 2\pi y$

$$\boxed{C_L(t) = 2\pi\left(-0.0425e^{-2t} - 0.0575e^{-0.26t} - 0.1e^{-2t}t + 0.1\right)\sigma(t)}$$



w vs t
Gust Velocity
Time
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
1
2
3
4
5
6
7
8
9
10

Cl vs. t