Problem U2 | Solution | Fall '03

from C.D.L.

a) $0.4\ \cancel{hp} \cdot 745.7\ W/\cancel{hp} = \boxed{298.3\ W}$

$298.3\ \cancel{W} / (1.356\ \cancel{W}/ft\text{-}lb/s) = \boxed{220.0\ ft\text{-}lb/s}$

b) heat flow $\dot{H} = 3 \cdot \text{Power} = 894.9\ W$

heat capacity of water $c = 4.2\ J/g\cdot{}^\circ K = 4200\ J/kg\cdot{}^\circ K$

typical body mass $m = 70\ kg$ (155 lb)

rate of temperature increase $\dot{T} = \frac{\dot{H}}{mc} = \frac{894.9\ W}{70\ kg \cdot 4200\ J/kg{}^\circ K} = 0.003{}^\circ K/s$

J/s

Human body can't tolerate more than a few degrees of temperature rise.

Say $\Delta T_{max} = 3^\circ K = \dot{T}\,\Delta t_{max}$

$\Rightarrow \Delta t_{max} = \frac{\Delta T_{max}}{\dot{T}} = \frac{3^\circ K}{0.003^\circ K/s} = \boxed{1000\ s} = \boxed{16.7\ \text{minutes}}$

c) dimensions, using SI units for example:

$\rho \sim kg/m^3$, $V \sim m/s$, $S \sim m^2$, $c \sim m$

$L \sim N = kg \cdot m/s^2$ (force), $M \sim N\text{-}m = kg\ m^2/s^2$ (moment)

equation: $L = \frac{1}{2} \rho V^2 S\, C_L$

units → $\underbrace{kg\ m/s^2}_{\text{same}} \sim (kg/m^3)(m/s)^2\, m^2\, C_L \sim \underbrace{kg\ m/s^2}_{\text{same}}\, C_L$

so $\boxed{C_L \text{ is dimensionless}}$

equation: $M = \frac{1}{2} \rho V^2 S\, c\, C_M$

units → $\underbrace{kg\text{-}m^2/s^2} \sim (kg/m^3)(m/s)^2\, m^2 \cdot m\, C_M \sim \underbrace{kg\ m^2/s^2}\, C_M$

so $\boxed{C_M \text{ is dimensionless}}$

d) geometric dimensions scaled by 1/2, with same airflow

$\rho \to \rho$ same, $V \to V$ same, $S \to \frac{1}{4} S$, $c \to \frac{1}{2} c$, C_L, C_m same

so $\boxed{L \to \frac{1}{4} L}$, $\boxed{M \to \frac{1}{8} M}$