Fluids – Lecture 16 Notes

1. Shock Losses
2. Compressible-Flow Pitot Tube

Reading: Anderson 8.6, 8.7

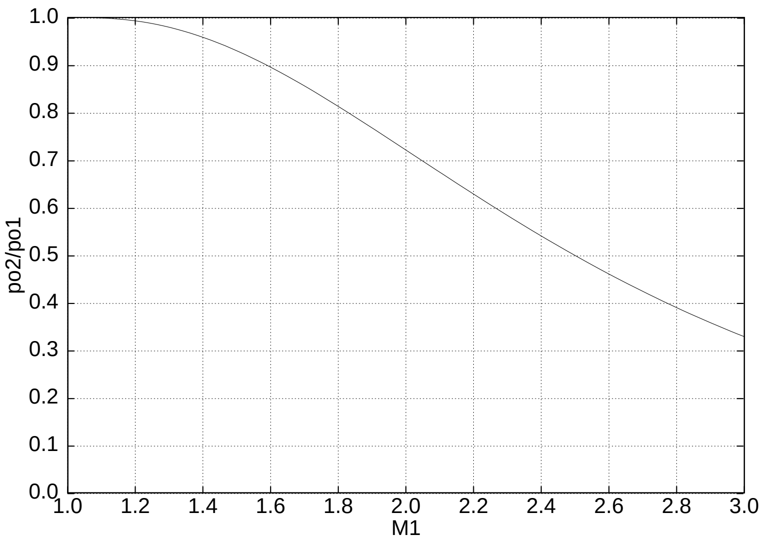
Shock Losses

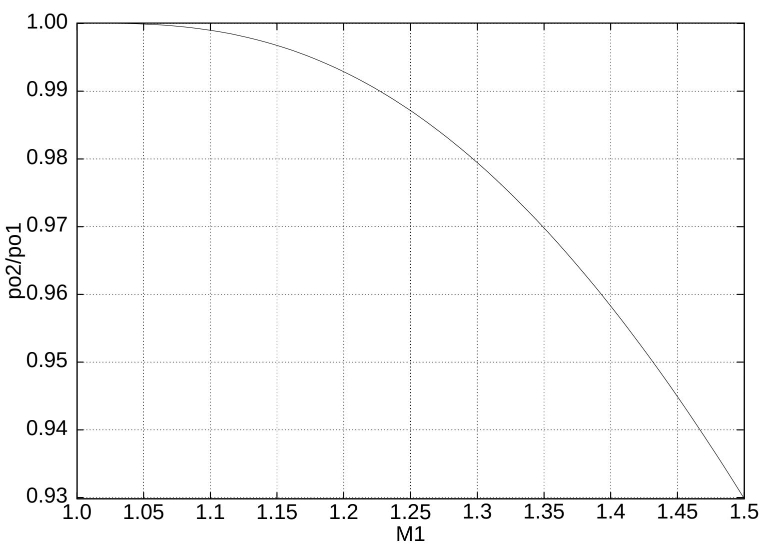
Stagnation pressure jump relation

The stagnation pressure ratio across the shock is

$$\frac{p_{o_2}}{p_{o_1}} = \frac{p_2}{p_1} \left(\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_1^2} \right)^{\gamma/(\gamma-1)} \tag{1}$$

where both p_2/p_1 and M_2 are functions of the upstream Mach number M_1, as derived previously. The figures show the p_{o_2}/p_{o_1} ratio, with the second figure showing an expanded scale near $M_1 \simeq 1$.





The fractional shock total-pressure loss $1 - p_{o_2}/p_{o_1}$ is seen to be small for M_1 close to unity, but increases rapidly for higher Mach numbers. Minimizing this loss is of great practical importance, since it cuts directly into the performance of supersonic air-breathing engines.

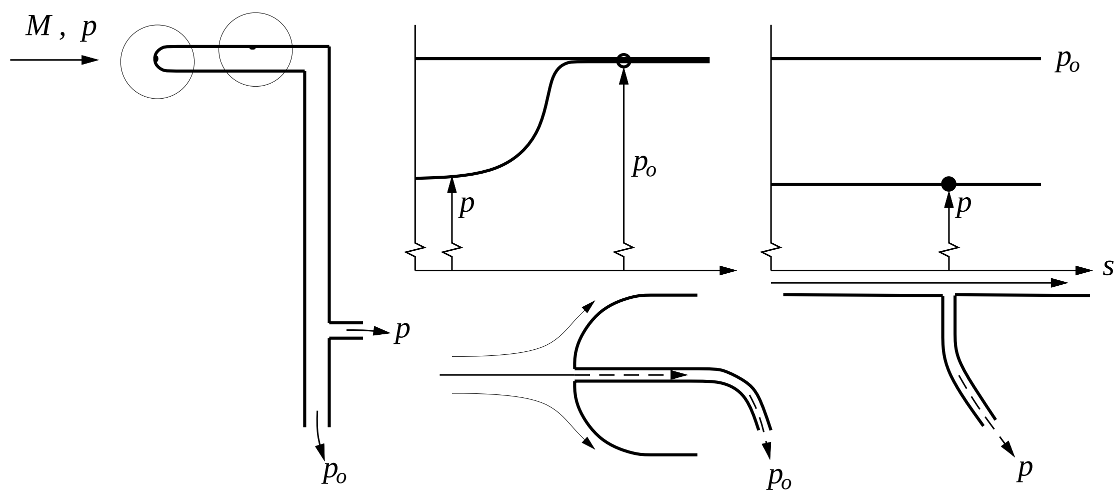
Compressible-Flow Pitot Tube

Subsonic pitot tube

A pitot tube in subsonic flow measures the local total pressure p_o. Together with a measurement of the static pressure p, the Mach number can be computed from the p_o/p ratio relation.

$$\frac{p_o}{p} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

$$M^2 = \frac{2}{\gamma-1}\left[\left(\frac{p_o}{p}\right)^{(\gamma-1)/\gamma} - 1\right] \qquad (2)$$



The pitot-static combination therefore constitutes a *Mach meter.* With M^2 known, we can then also determine the dynamic pressure.

$$\frac{1}{2}\rho V^2 = \frac{\gamma}{2} p M^2 = \frac{\gamma}{\gamma-1} p \left[\left(\frac{p_o}{p}\right)^{(\gamma-1)/\gamma} - 1\right] \qquad (3)$$

The velocity can be determined from

$$V^2 = a^2 M^2 = \frac{a_o^2 M^2}{1 + \frac{\gamma-1}{2} M^2}$$

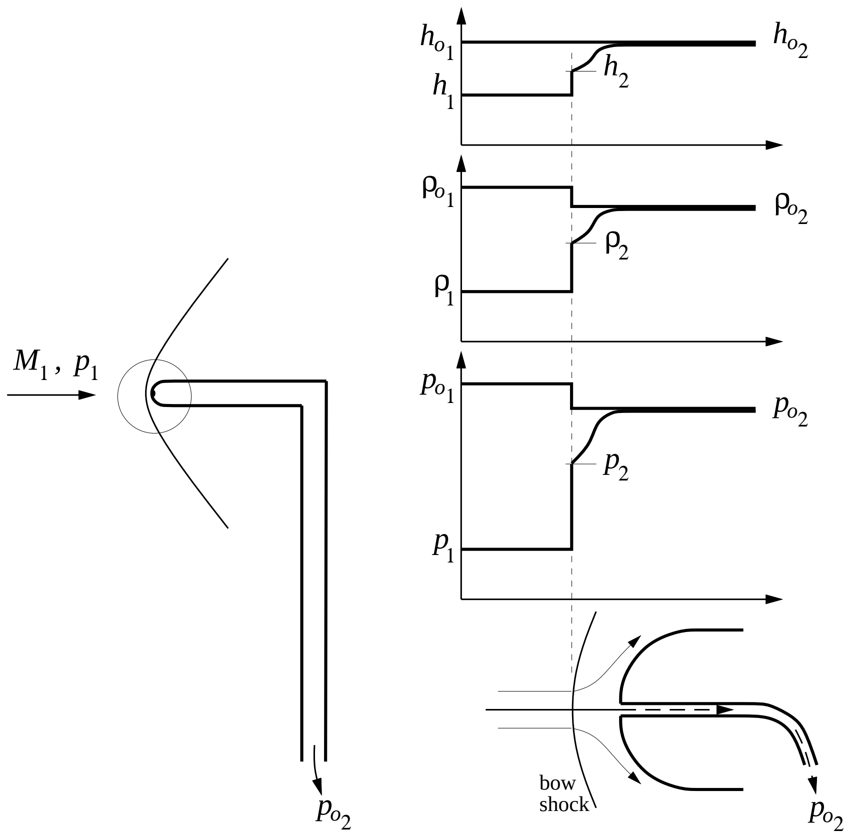
but this requires knowing either the static speed of sound a, or the stagnation speed of sound a_o. The latter can be obtained by measuring the stagnation temperature at the tip of the pitot probe.

Supersonic pitot tube

A pitot probe in a supersonic stream will have a *bow shock* ahead of it. This complicates the flow measurement, since the bow shock will cause a drop in the total pressure, from p_{o_1} to p_{o_2}, the latter being sensed by the pitot port. It's useful to note that the shock will also cause a drop in ρ_o, but h_o will not change.

The pressures and Mach number immediately behind the shock are related by

$$\frac{p_{o_2}}{p_2} = \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\gamma/(\gamma-1)}$$



In addition, we also have M_2 and p_2/p_1 as functions of M_1 from the earlier normal-shock analysis. Combining these produces the relation between the p_{o_2} measured by the pitot probe, the static p_1, and the required flow Mach number M_1. After some manipulation, the result is the *Rayleigh Pitot tube formula*

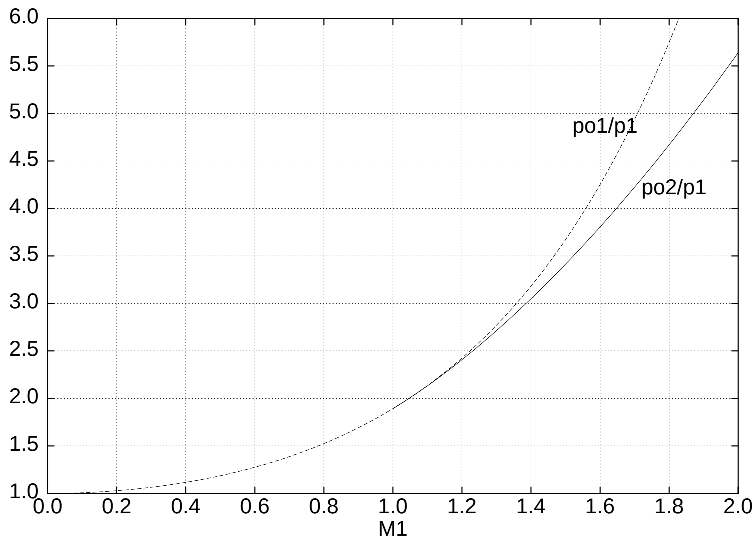
$$\frac{p_{o_2}}{p_1} = \frac{p_{o_2}}{p_2}\frac{p_2}{p_1} = \left(\frac{(\gamma+1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma-1)}\right)^{\gamma/(\gamma-1)} \frac{1-\gamma+2\gamma M_1^2}{\gamma+1} \tag{4}$$

The figure shows p_{o_2}/p_1 versus M_1, compared with the isentropic ratio p_{o_1}/p_1.

$$\frac{p_{o_1}}{p_1} = \left(1 + \frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)} \tag{5}$$

Only the latter is plotted for $M_1 < 1$, where there is no bow shock, and so equation (4) does not apply. The effect of the pitot bow shock's total pressure loss, indicated by the difference $p_{o_1} - p_{o_2}$, becomes substantial at larger Mach numbers.

Ideally, we would like to have the pitot formula (4) give M_1 as an explicit function of the pitot/static pressure ratio p_{o_2}/p_1. However, this is not possible due to its complexity, so a numerical solution is required. The function is also readily available in table form.



6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
M1
po1/p1
po2/p1