Lecture S9 Muddiest Points

General Comments

In this lecture, we began learning how to find the time response of RC circuits to initial conditions. To do this, we used the node method, and used the constitutive relations for capacitors and resistors. This led to coupled, first order differential equations, which we can solve by guessing that the solution is exponential, proportional to e^{st}. We then find that solutions are possible only for some exponentials. This gives the characteristic values, s, of the system.

Responses to Muddiest-Part-of-the-Lecture Cards

(57 cards)

1. ***Why do we assume that the guess we made (e^{st}) is right and go ahead as if it could not be wrong? (1)*** We assume the exponential as a trial answer, and show that it indeed works. That is, the trial solution does indeed solve the differential equations, and you can choose constants (the characteristic vector) to solve the initial conditions. It *could* have been wrong, and we would have found out if it were.

2. ***When doing row reduction, can you alter columns at all? (1) The row reduction approach seems to have no rules, i.e., you can do anything to the rows.*** The legal row operations are (1) scaling of a row; (2) exchanging rows; and (3) adding the multiple of one row to another row. You can't do column operations.

3. ***Why is it necessary to do row reduction when solving for the [characteristic vectors]? (1)*** Row reduction is the most direct way to find solutions to the equation

$$M(s)\underline{E} = \underline{0} \tag{8}$$

4. ***When solving for characteristic vectors, why can we choose an arbitrary value for E_3? (2)*** If the characteristic vector $\underline{E}$ is a solution to $M(s)\underline{E} = \underline{0}$, then so is any multiple of the characteristic vector. So unless it turns out that $E_3 = 0$, E_0 can be arbitrarily chosen. Indeed, we must make some arbitrary choice, since the characteristic vector is not unique.

5. ***I don't remember any 18.03. (1) I am so lost without 18.03. (1)*** I hope you're not too lost — I'm covering all the material that you should need.

6. ***Can the characteristic values be obtained by solving the matrix using eigenvectors? (1)*** Not exactly. For eigenvalues to be appropriate, you must have the differential equations is in a specific form. We will do that, but not yet.

7. ***How are characteristic values / vectors different from eigenvalues / vectors? (3)*** Characteristic values and eigenvalues are really the same thing. Characteristic vectors and eigenvectors are different — only when the differential equations are in a specific form does it make sense to talk about eigenvectors. The ideas are very similar, though.

8. ***Why is s related to d/dt? (1)*** When we assume solutions of the form, say, $e_1(t) = E_1 e^{st}$, then the time derivative of $e_1(t)$ becomes

$$\frac{d}{dt} e_1(t) = s E_1 e^{st} \tag{9}$$

That is, each d/dt in the differential equation becomes s.

9. ***Could you clarify trivial and nontrivial solutions. (1)*** We're solving the equation

$$M \underline{E} = \underline{0} \tag{10}$$

If M is a square matrix, one obvious solution is $\underline{E} = \underline{0}$. This is the "trivial solution." But we're looking for a non-zero solution, so this doesn't help very much. We need a *nontrivial* solution, that is, one that is nonzero. This is only possible if $\det(M) = 0$.

10. ***Why is capacitance Cd/dt? (1)*** It isn't! Cd/dt for a capacitor plays a role similar to conductance for a resistor.

11. ***Are the solutions to this type of problem always in eigenvector form? (1)*** Essentially, yes. The general solution to a system described by a homogenous, linear, time-invariant differential equations can always be solved as an eigenvector problem.

12. ***When you do row reduction, why do you change the pivots to 1 first? (1)*** You don't have to, but it makes the process easier.

13. ***Just need row reduction practice. (1) My only mud was the row reduction method. (1)*** Many of the problem set problems will require row reduction as one step, so you should get some practice.

14. ***So to get a non-trivial solution, we have to set the admittance to $s = -1$ or $s = -1/3$ How do we do that? (1)*** You must have misheard me. s is not the admittance, it's the characteristic value. The admittance is Cs. It's not that we set the admittance — the issue that there is a solution to the equations only for specific values of s.

15. ***In the example, when $s = -1$, you can pick any one of the three E_1, E_2, E_3 to be whatever you want it to be? Or only one of E_1 and E_3 to be whatever you want? (1)*** The letter. In the example, $E_2 = 0$, and it cannot be set to be anything else. After row reduction, the second row gives

$$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = 0 \tag{11}$$

which implies $E_2 = 0$.

16. ***How did you find a and b in the last example? (2)*** We found that the general solution is

$$a \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} e^{-t} + b \begin{bmatrix} 1 \\ 2/3 \\ 1 \end{bmatrix} e^{-t/3} \tag{12}$$

The initial conditions are the voltages across the capacitors, so that

$$\begin{aligned} v_1(0) = &\quad e_1 = -a + b \\ v_5(0) = &\quad e_3 = a + b \end{aligned} \tag{13}$$

17. ***Could you refresh my memory on how this is transformed in to a* basis*? (1)*** Please see me about this. The explanation is a little longer than I can answer is this forum.

18. ***Do you think you could give us a few examples to try on our own? (1)*** Please see the archives on the Unified website for old problem sets.

19. ***How well do we need to know tools such as gaussian reduction? Should we know all methods on [unreadable]? (1)*** You should know how to solve linear equations to the linear of the linear algebra primer.

20. ***Will we be larger than matrices larger than 3×3? (1)*** You should understand how to deal with general matrices.

21. ***How can you decide that the terms like $C_1 \frac{d}{dt} + G_2$ aren't related to time? They have $\frac{d}{dt}$ in them. (1)*** Yes, but $\frac{d}{dt}$ is a time-invariant (or shift-invariant) operator. Said another way, $\frac{d}{dt}$ is not a function of time, it's an operator on functions of time.

22. ***Not clear how you found $C_1 \frac{d}{dt}$ is the admittance of a capacitor. (1)*** The current through a resistor is v/R, or Gv. So $G = 1/R$ is the conductance of the resistor. The current through a capacitor is $C\frac{dv}{dt}$. By analogy, $C\frac{d}{dt}$ is the conductance (which we call admittance) of the capacitor.

23. ***How do you get the answer after you've reduced a row? (1)*** By back substitution. Please see the primer.

24. ***You started to explain the method's relationship to using Laplacians to solve this. What is it? (1)*** Actually, I said that what we are doing is related to Laplace transforms. We'll go into this in great detail next term.

25. ***No mud. (27)*** Good!