Department of Aeronautics & Astronautics, M.I.T.
16.001 - Materials & Structures

Quiz No. 4

Instructor: Raúl Radovitzky

Student's name:______________________________

Question	Points	Score
1	15	
2	15	
Total:	30	

Letter grade: __________

Question 1 [15 points]

It is shown in higher-level classes on elasticity theory that the stress field near a crack tip, as shown in Figure 1, subject to remote stresses is dominated by the following expressions for the stress components in cylindrical coordinates (r, θ) where r is the distance to the crack tip, θ is the angle sketched in the figure, and K is the so-called *stress intensity factor*, which is determined by the geometry of the domain and the remote loading conditions.

$$\sigma_{rr}(r,\theta) = \frac{K}{\sqrt{2\pi}} r^{-1/2} \left[\frac{5}{4} \cos\left(\frac{\theta}{2}\right) - \frac{1}{4} \cos\left(\frac{3\theta}{2}\right) \right] \tag{1}$$

$$\sigma_{\theta\theta}(r,\theta) = \frac{K}{\sqrt{2\pi}} r^{-1/2} \left[\frac{3}{4} \cos\left(\frac{\theta}{2}\right) + \frac{1}{4} \cos\left(\frac{3\theta}{2}\right) \right] \tag{2}$$

$$\sigma_{r\theta}(r,\theta) = \frac{K}{\sqrt{2\pi}} r^{-1/2} \left[\frac{1}{4} \sin\left(\frac{\theta}{2}\right) + \frac{1}{4} \sin\left(\frac{3\theta}{2}\right) \right] \tag{3}$$

Note that the given stress components are in the basis $\mathbf{e}_r, \mathbf{e}_\theta$, which changes at each point (r, θ), and the functions are expressed in terms of cylindrical coordinates r, θ.

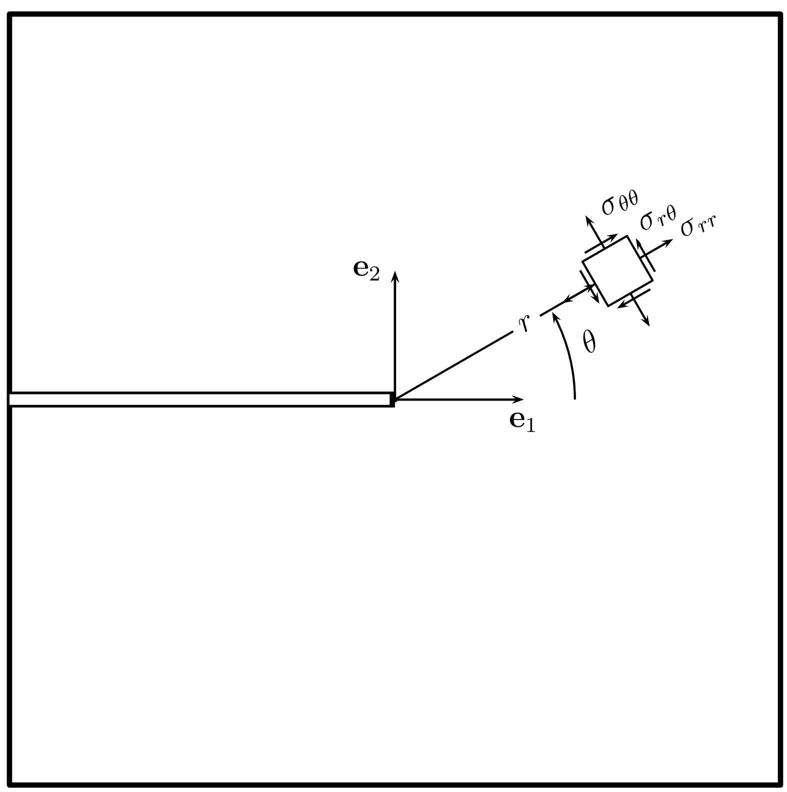

Figure 1:

1.1 (5 points) Use appropriate equations of stress equilibrium from the notes to show that this stress field is in equilibrium. Justify your choice of the specific version of the equilibrium equations.

1.2 (5 points) Among the peculiarities of this stress field, show that the principal directions of stress at any point in the domain are given by:

$$\tan(2\alpha_p) = \cot(\theta/2)$$

From this expression, use the trigonometric identity $\cot x = \tan(\pi/2 - x)$ to show that:

$$\alpha_p = \begin{cases} (\pi - \theta)/4 & 0 \leq \theta \leq \frac{\pi}{2} \\ (-\pi - \theta)/4 & -\frac{\pi}{2} \leq \theta \leq 0 \end{cases}$$

1.3 (5 points) Show that the state of stress along the line $\theta = 0$ is purely hydrostatic and the normal stress components on that line are $\sigma_{11} = \sigma_{22} = \frac{K}{\sqrt{2\pi r}}$

Question 2 [15 points]

(M.O. M11)

The state of strain at a point in an aluminum component of the fuselage of an airplane is measured with a delta strain gauge rosette (See Figure 2, where each gauge is a side of an equilateral triangle) of three strain gauges a, b, c. The strain gauges read $\epsilon_a = 15 \times 10^{-6}, \epsilon_b = 60 \times 10^{-6}, \epsilon_c = 80 \times 10^{-6}$.

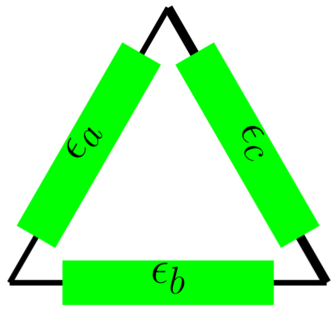

Figure 2: Delta Rosette strain gauge

Determine:

2.1 (5 points) All the components of strain in cartesian axes $\mathbf{e}_1, \mathbf{e}_2$ respectively aligned with the horizontal and vertical direction. Clearly indicate the basis vectors and their angles with respect to the each other that you use in your calculations.

2.2 (5 points) The principal strains $\epsilon_{I,II}$, their directions $\alpha_{I,II}$

2.3 (5 points) The maximum shear strains γ^{max} and their directions α_s

End of Test