16.001 - Materials & Structures
Problem Set #8

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Question	Points
1	5
2	5
3	8
4	3
5	3
Total:	24

○ **Problems M-8.1** [5 points]

You've been tasked with selecting the material for the grid fins on SpaceX's next launch vehicle, Starship. These grid fins will be significantly larger than the ones on the Falcon 9 (7x3 m^2 vs. 2x1.2 m^2), making cost a much bigger concern. The grid fins should be light, cheap, and capable of surviving multiple exposures to high temperatures (>400 °C). They also need to be stiff so that they don't deflect during reentry.

In each of the following materials selection problems, list the function, objective(s), constraints, and materials indices with which ranked the different materials. You can use this reference to determine the appropriate materials index. Show the Property Diagram(s) that you used to make your decisions with the appropriate materials index contour overlaid. Indicate on the property diagram the best 2 or 3 materials options using the labeling function.

○ **Problems M-8.2** [5 points]
The extensional and shear strains at a point of a loaded structure have been measured with respect to a particular set of cartesian basis vectors. The measured values are

$$\epsilon_{11} = -800 \times 10^{-6} \quad (1)$$

$$\epsilon_{22} = -200 \times 10^{-6} \quad (2)$$

$$\gamma_{12} = -600 \times 10^{-6} \quad (3)$$

2.1 (1 point) Draw Mohr's circle for this state of strain

2.2 (2 points) Find the principal strains and principal directions. Show also the deformed shape of an element which originally was a parallelepiped with its faces parallel to these axes

2.3 (2 points) Find the maximum shear strains and corresponding directions. Show also the deformed shape of an element which originally was a parallelepiped with its faces parallel to these axes

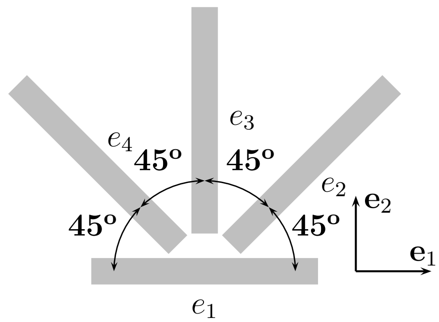

Figure 1: T-V rosette strain gauge

○ **Problems M-8.3** [8 points]

Consider the T-V rosette shown in Figure 1. The measured strains along the directions of the individual strain gauges are respectively $e_1 = 910\mu$, $e_2 = 990\mu$, $e_3 = 310\mu$, and $e_4 = 190\mu$.

3.1 (2 points) Use the equations of transformation of strain components in 2D as many times as needed, to relate the measured strain components and those in the cartesian system $\mathcal{E} = (\mathbf{e}_1, \mathbf{e}_2)$

3.2 (2 points) Can you determine the strain components $\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}$ from these equations? Do you have insufficient or redundant information? How can this be useful from the experimental standpoint?

3.3 (3 points) Use a least-squares approach to obtain the "best approximation" to the strain components $\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}$ in terms of the measured data. Hint: as it name indicates, the least squares method finds a solution of the overdetermined system by minimizing the sum of the square of the errors incurred in the satisfaction of each equation.

3.4 (1 point) Find the orientation of the principal strain directions, and the principal strains

○ **Problems M-8.4** [3 points]
(M.O. M11)

The state of strain in a composite is determined by a rectangular strain gauge rosette attached to the surface, as shown in Figure 2. The three strain gauges (a, b, & c) are arranged at angles $\alpha_a = 0^o, \alpha_b = 45^o, \&\ \alpha_c = 90^o$. The gauges read $\epsilon_a = 20 \times 10^{-6}, \epsilon_b = 55 \times 10^{-6}, \epsilon_c = -60 \times 10^{-6}$. The composite is a polymer matrix reinforced with unidirectional fibers that are aligned at 120^o from horizontal.

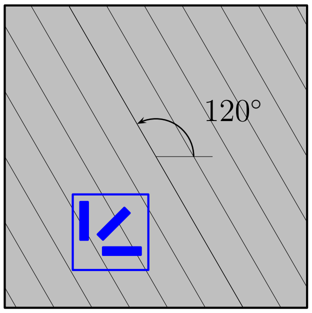

Figure 2: Composite material with 3 strain gauges

Determine the normal and shear strain components in the directions aligned and perpendicular to the fibers.

◯ **Problems M-8.5** [3 points]
(M.O. M11)
The state of strain at a point in an aluminum component of the fuselage of an airplane is measured with a delta strain gauge rosette (See Figure 3, where each gauge is a side of the triangle) of three strain gauges a, b, c arranged at angles $\alpha_a = 0, \alpha_b = 60, \alpha_c = 120°$. The strain gauges read $\epsilon_a = 15 \times 10^{-6}, \epsilon_b = 60 \times 10^{-6}, \epsilon_c = 80 \times 10^{-6}$.

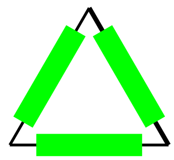

Figure 3: Delta Rosette strain gauge

Determine:

5.1 (1 point) All the components of strain in cartesian axes $\mathbf{e}_1, \mathbf{e}_2$ respectively aligned with the horizontal and vertical direction

5.2 (1 point) The principal strains $\epsilon_{I,II}$, their directions $\alpha_{I,II}$

5.3 (1 point) The maximum shear strains γ^{max} and their directions α_s