# 16.001 - Materials & Structures
# Problem Set #2

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○ **Problem M-2.1**

Hardness testing is a simple, fast, and non-destructive way to assess the yield strength of a material, i.e., the stress at which a material starts to deform plastically, which corresponds to the onset of non-linearity in the stress-strain or load-displacement curve. Hardness testing can probe small volumes of materials (e.g., nanoindentation hardness testing) and can be applied to brittle materials, such as ceramics, that are difficult to test using conventional uniaxial compression or tension testing. There are many different variants of the hardness test (Rockwell, Meyer, Knoop, Vickers, nanoindentation, etc.), but in all these different variants, an indenter (often sharp with a self-similar shape) is pressed into a specimen under a known load for a fixed time. The ratio of the load ($F$, units: N) to the projected area of the impression ($A$, units: $\mathrm{m}^{-2}$) gives the hardness ($H$, units: $\mathrm{N.m}^{-2}$ or Pa) which is related to the yield strength ($\sigma_y$) by $H \sim 3\sigma_y$. The origin of the proportionality constant of 3 is discussed on pg 105 of Ashby and Jones using a VERY elegant slip line argument.

In this problem, you are going perform a hardness test on a cheese (or any other food product) of your choice using a writing utensil (or any other sharp, nominally self-similar indenter).

1. Describe your experimental setup and test specimen.
2. Press the indenter into your specimen under a fixed load. You can use your hand to apply the load or some other object with known weight. For reference, an apple has a weight of 1 N or you can compute the weight of a known volume of water. Choose the load such that the indentation depth remains in the tip portion of the writing untensil and does not come too close to the cylindrical body. Apply the load for 3 s, measure the projected area of the indentation, and give the resulting hardness and strength in Pa.
3. How does this compare with the strength of common engineering materials (e.g., aluminum alloys, steels, ceramics, polymers) from the property diagram shown in Lecture 1?
4. How does the hardness vary with position throughout your cheese/food?
5. Repeat the hardness test but use different indentation times (1 s, 10 s, and 30 s). Summarize your measurements in a table. How does the hardness change with time? Is the cheese creeping (exhibiting time-dependent inelastic deformation)? Creep is critically important in high-temperature aerospace applications, like propulsion systems and thermal protection systems. Please upload a description of your specimen (cheese name, age, taste, test location (rind v. body)) as well as your experimental measurements to this : google spreadsheet

**Solution:**

### Problem M-2.2

(M.O. : M4) Figure 1 shows an airplane engine pod suspended from the wing. The engine has a weight of 11KN which acts at point G. The propeller turns clockwise when viewed from behind, and is delivering a thrust force of 17.5KN and a torque of 20KN·m.

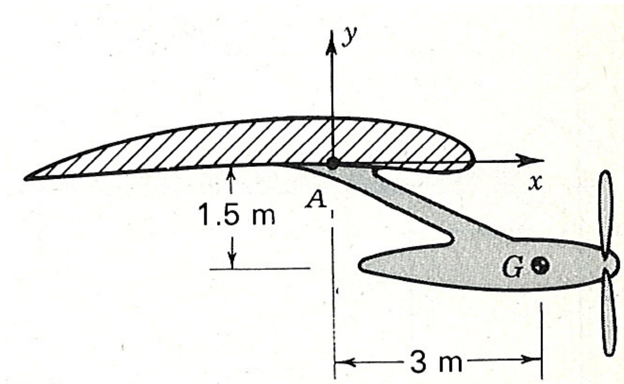


Figure 1: Suspended ariplane engine



What are the forces and moments exerted by the strut onto the wing at A? (3 pts)

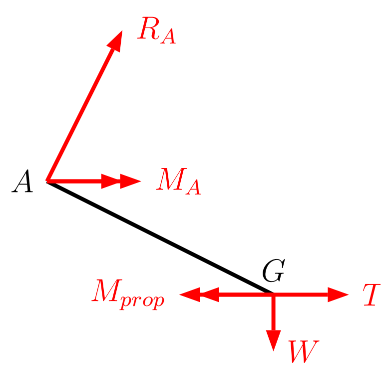


Figure 2: FBD of suspended engine

**Solution:** Start by drawing a FBD. From the FBD shown in Figure 2, lets write the equations of equilibrium.

$$\sum F_{e_1} = 0 = T + R_{A_1}$$
$$\sum F_{e_2} = 0 = R_{A_2} - W$$
$$\sum F_{e_3} = 0$$
$$\sum (M_A)_{e1} = 0 = M_{A_1} - M_{prop}$$
$$\sum (M_A)_{e2} = 0$$
$$\sum (M_A)_{e3} = 0 = M_{A_3} + T \cdot (1.5) - W \cdot (3)$$

where T, $R_A$, $M_A$, W, & $M_{prop}$ are the thrust, force reaction, moment reaction at the support, weight, and moment applied by the engine, respectively.Simplifying the equations yield:

$$R_{A_1} = -T = 17.5KN$$
$$R_{A_2} = W = 11KN$$
$$R_{A_3} = 0KN$$
$$M_{A_1} = M_{prop} = 20KN \cdot m$$
$$M_{A_2} = 0KN \cdot m$$
$$M_{A_3} = 6.75KN \cdot m$$

2020

### Problem M-2.3

A four-engine jet transport, which weighs 230,000 lb fully loaded, has its center of gravity at the location shown in Figure 3. Before taking off, the pilots must test the engines by operating them, one at a time at a thrust of 8,000 lb. As they check the left outboard engine, the other three engines idle producing no thrust. The rear-wheel brakes are locked during the test, but the nose wheel has no brakes. In addition, the nose wheel is mounted on a caster, so it cannot resist any sidewise force.

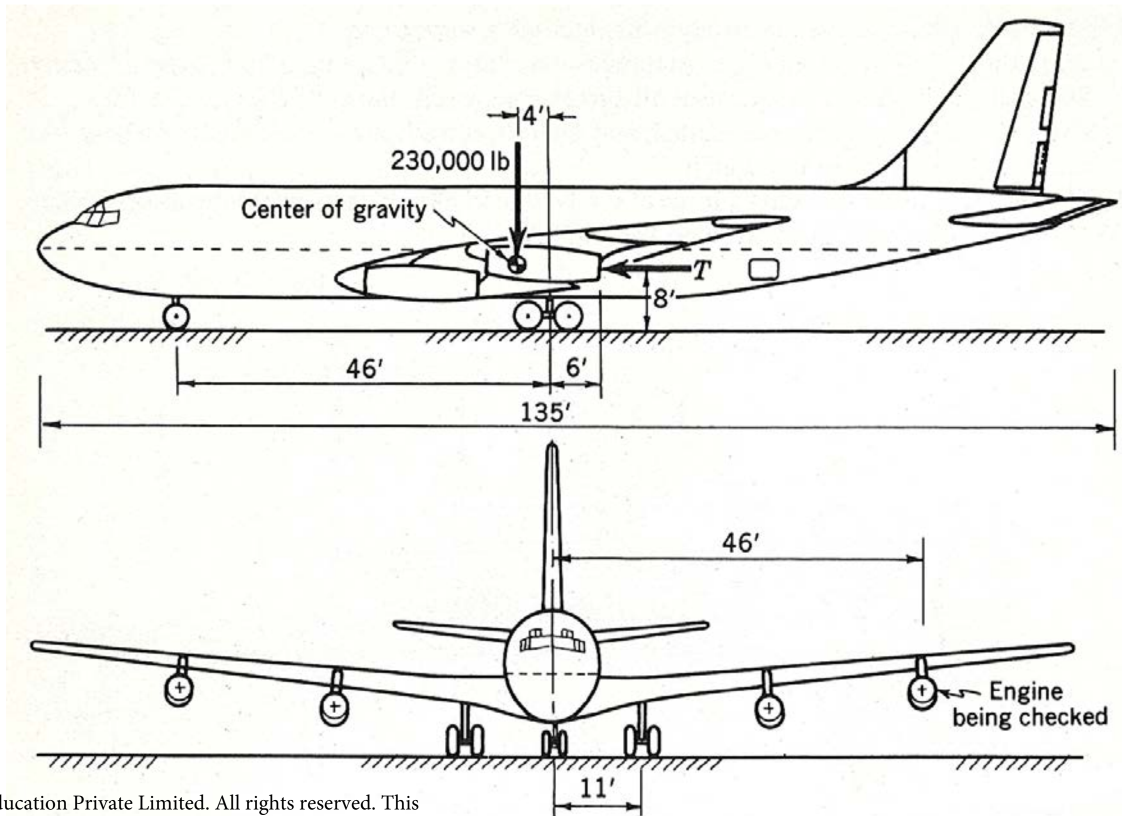




Figure 3: The considered four-engine jet.

(a) What forces does the ground exert on the landing wheels during the test? (3 pts)

**Solution:**
Start by setting a coordinate system relative to the center of gravity. The system used in this solution $e_1$ points to the nose of the jet, $e_2$ points out the jet in the direction of the left wing, and $e_3$ points toward the ceiling of the jet. Next, let's write all the forces in their component forms along with needed position vectors.

$$\boldsymbol{F_{BL}} = \begin{bmatrix} F_{BL_1} \\ 0 \\ F_{BL_3} \end{bmatrix}, \ \boldsymbol{F_{BR}} = \begin{bmatrix} F_{BR_1} \\ 0 \\ F_{BR_3} \end{bmatrix} \ \boldsymbol{F_F} = \begin{bmatrix} 0 \\ 0 \\ F_{F_3} \end{bmatrix}$$

$$\boldsymbol{T} = \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{W} = \begin{bmatrix} 0 \\ 0 \\ W \end{bmatrix}, \boldsymbol{r_E} = \begin{bmatrix} -10 \\ 46 \\ 0 \end{bmatrix}$$

$$\boldsymbol{r_F} = \begin{bmatrix} 42 \\ 0 \\ -8 \end{bmatrix}, \boldsymbol{r_{BL}} = \begin{bmatrix} -4 \\ 11 \\ -8 \end{bmatrix}, \boldsymbol{r_{BR}} = \begin{bmatrix} -4 \\ -11 \\ -8 \end{bmatrix}$$

where $F_{BL}$ & $F_{BR}$ refer to the left and right back wheels. $F_F$ is the reaction on the front wheel. The position vectors, $r$, are measured from the center of gravity. E, BL, BR, F correspond to the locations engine, back left wheel, back right wheel, and front wheel, respectively. Now, write the equations of equilibrium

$$\sum \boldsymbol{F} = \boldsymbol{F_{BL}} + \boldsymbol{F_{BR}} + \boldsymbol{F_F} + \boldsymbol{T} + \boldsymbol{W} = \boldsymbol{0}$$

$$\begin{bmatrix} F_{BL_1} \\ 0 \\ F_{BL_3} \end{bmatrix} + \begin{bmatrix} F_{BR_1} \\ 0 \\ F_{BR_3} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ F_{F_3} \end{bmatrix} + \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -W \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sum \boldsymbol{M_O} = \boldsymbol{r_E} \times \boldsymbol{T} + \boldsymbol{r_F} \times \boldsymbol{F_F} + \boldsymbol{r_{BL}} \times \boldsymbol{F_{BL}} + \boldsymbol{r_{BR}} \times \boldsymbol{F_{BR}}$$

$$\begin{bmatrix} 0 \\ 0 \\ -46T \end{bmatrix} + \begin{bmatrix} 0 \\ -42F_{F_3} \\ 0 \end{bmatrix} + \begin{bmatrix} 11F_{BL_3} \\ 4F_{BL_3} - 8F_{BL_1} \\ -11F_{BL_1} \end{bmatrix} + \begin{bmatrix} -11F_{BR_3} \\ 4F_{BR_3} - 8F_{BR_1} \\ 11F_{BR_1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving the system of equations, we get the forces on the wheels.
Back left wheel:

$$\begin{aligned} \boldsymbol{F_{BL}} &= -\frac{57}{22}Te_1 + \left(-\frac{1}{23}T + \frac{21}{46}W\right)e_3 \\ &= -20727.3lb\ e_1 + 104652.2lb\ e_3 \end{aligned}$$

Back right wheel:

$$\begin{aligned} \boldsymbol{F_{BR}} &= \frac{35}{22}Te_1 + \left(-\frac{1}{23}T + \frac{21}{46}W\right)e_3 \\ &= 12727.3lb\ e_1 + 104652.2lb\ e_3 \end{aligned}$$

Front wheel:

$$\boldsymbol{F_F} = \frac{2}{23}(W + T)e_3 = 20695.7lb\ e_3$$

(b) What must the coefficient of friction between the ground and the wheels be to prevent the rear wheels from slipping? (2 pts)

**Solution:** Since the back wheels both have the same normal force ($e_3$-force) on them look for the wheel with the largest force in the $e_1$ direction to determine the coefficient. The largest magnitude of force in the $e_1$ direction is found in the left wheel.

$$F_{fric} \leq \mu F_N$$
$$F_{BL_1} = \mu F_{BL_3}$$
$$\rightarrow \mu \geq \frac{F_{BL_1}}{F_{BL_3}} = \frac{20727.3}{104652.2} = 0.198$$

2020

## Problem M-2.4

Figure 4 shows the external forces acting on a version of a supersonic transport (SST) just prior to touchdown. The dimensions are included in the figure below it. The following information is given:

(1) The "Canard" (forward) control surface is set at its zero-lift angle of attack, and causes a drag force of 50 N.

(2) The Aircraft weighs 200,000 N.

(3) The aerodynamic moments about $AC_c$, $AC_w$, & $AC_t$ can be neglected.

(4) The lift-to-drag ratio for the tail is given by $(L/D)_t = 1.8$.

(5) The aircraft produces a thrust of 80,000 N.

(6) The aircraft is in static equilibrium.

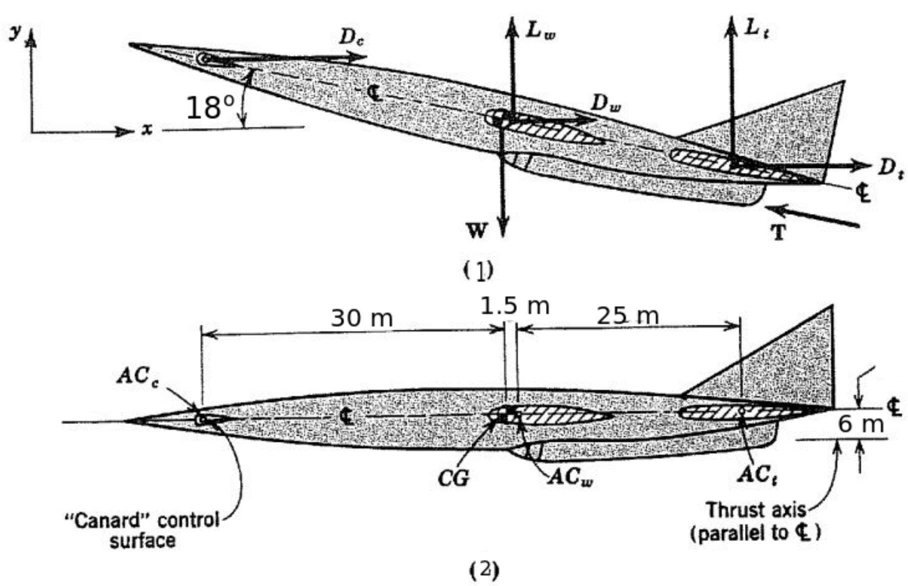


Figure 4: Forces on an SST prior to touchdown



With this information, determine the values of the tail lift $L_t$, the tail drag $D_t$, the lift of the wing $L_w$, and the drag of the main wing $D_w$ (3 pts)

**Solution:**

We begin by redrawing the free-body diagram(FBD) of the aircraft above. Through looking at the FBD we can write the equations of equilibrium for the aircraft. Three equation are required to solve the problem (Forces in x and y & momentum about the z axis).

$$\sum F_x = D_c + D_w + D_t - T\cos(\alpha) \tag{1}$$

$$\sum F_y = L_w + L_t + T\sin(\alpha) - W \tag{2}$$

$$\begin{aligned}\sum M_{cg} = -D_c x_{cg-c}\sin(\alpha) + L_w x_{cg-w}\cos(\alpha) + D_w x_{cg-w}\sin(\alpha) \\ + L_t x_{cg-t}\cos(\alpha) + D_t x_{cg-t}\sin(\alpha) - T x_{cg-thrust}\end{aligned} \tag{3}$$

where $\alpha$ = Angle of Attack, $x_{cg-c}$ is the distance between the center of gravity(cg) and the canard, $x_{cg-w}$ is the distance between the cg and the aerodynamic center of the wing, $x_{cg-t}$ is the distance between the cg and the aerodynamic center of the tail, and $x_{cg-thrust}$ is the distance between the centerline and the axis of thrust.

By substituting the lift-to-drag ratio of the tail and knowing that $\sum F_x = 0$, $\sum F_y = 0$, & $\sum M_{cg} = 0$, the system of equations becomes:

$$0 = D_c + D_w + D_t - T\cos(\alpha) \tag{4}$$

$$0 = L_w + (L/D)_t D_t + T\sin(\alpha) - W \tag{5}$$

$$\begin{aligned}0 = -D_c x_{cg-c}\sin(\alpha) + L_w x_{cg-w}\cos(\alpha) + D_w x_{cg-w}\sin(\alpha) \\ + (L/D)_t D_t x_{cg-t}\cos(\alpha) + D_t x_{cg-t}\sin(\alpha) - T x_{cg-thrust}\end{aligned} \tag{6}$$

This is now a solvable system of equations, resulting in:

$D_w = 72200N$

$L_w = 168000N$

$D_t = 3860N$

By plugging $D_t$ back into the lift-to-drag ratio of the tail the final unknown value is calculated to be:

$L_t = 6950N$

2019, 2020

**Problem M-2.5**

The three pipes in Figure 5 are rigidly joined and clamped at the origin. The other end of the pipe assembly is located at point **A** with coordinates $\{3, 2, 2\}$m. The forces are given by the vectors: $\mathbf{F}^{(1)} = (-2\text{kN})\mathbf{e}_1 + (-1\text{kN})\mathbf{e}_2 + (-1\text{kN})\mathbf{e}_3$, $\mathbf{F}^{(2)} = (-1\text{kN})\mathbf{e}_1 + (1\text{kN})\mathbf{e}_2 + (1\text{kN})\mathbf{e}_3$

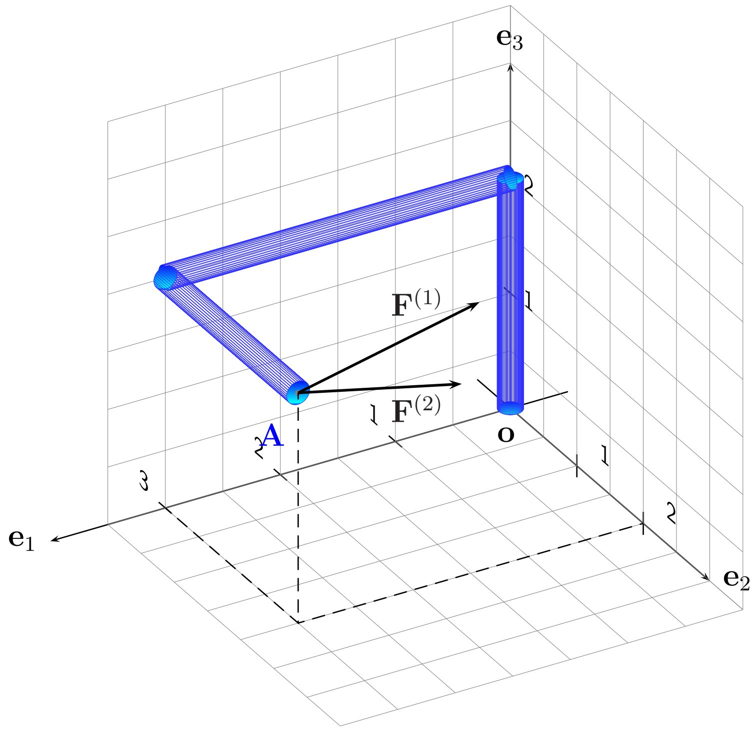


Figure 5: Pipe assembly

Find the reaction force and moment vectors at the origin (3 pts)

**Solution:** Following the free body diagram in Figure 6:

Equilibrium of forces:

$$\mathbf{F}^{(1)} + \mathbf{F}^{(2)} + \mathbf{R}^o = \mathbf{0}$$

$$(-2\text{kN} - 1\text{kN} + R_1^o)\mathbf{e}_1 + (-1\text{kN} + 1\text{kN} + R_2^o)\mathbf{e}_2 + (-1\text{kN} + 1\text{kN} + R_3^o)\mathbf{e}_3 = (0\text{kN})\mathbf{e}_1 + (0\text{kN})\mathbf{e}_2 + (0\text{kN})\mathbf{e}_3$$

$$\boxed{R_1^o = 3\text{kN}, R_2^o = 0, R_3^o = 0, \mathbf{R}^o = 3\text{kN}\mathbf{e}_1}$$

Equilibrium of moments: Take moments with respect to **o** so that the reaction does not parcipate in the equation

$$\mathbf{r}^A \times \mathbf{F}^{(1)} + \mathbf{r}^A \times \mathbf{F}^{(2)} + \mathbf{M}^o = \mathbf{0}$$

$$(3\text{m}\mathbf{e}_1 + 2\text{m}\mathbf{e}_2 + 2\text{m}\mathbf{e}_3) \times (-3\text{kN})\mathbf{e}_1 + M_1^o\mathbf{e}_1 + M_2^o\mathbf{e}_2 + M_3^o\mathbf{e}_3 = (0\text{kNm})\mathbf{e}_1 + (0\text{kNm})\mathbf{e}_2 + (0\text{kNm})\mathbf{e}_3$$

$$(0\text{kNm} + M_1^o)\mathbf{e}_1 + (6\text{kNm} + M_2^o)\mathbf{e}_2 + (6\text{kNm} + M_3^o)\mathbf{e}_3 = (0\text{kNm})\mathbf{e}_1 + (0\text{kNm})\mathbf{e}_2 + (0\text{kNm})\mathbf{e}_3$$

which results in:

$$\mathbf{M}^o = (-6\text{kNm})\mathbf{e}_2 + (-6\text{kNm})\mathbf{e}_3$$

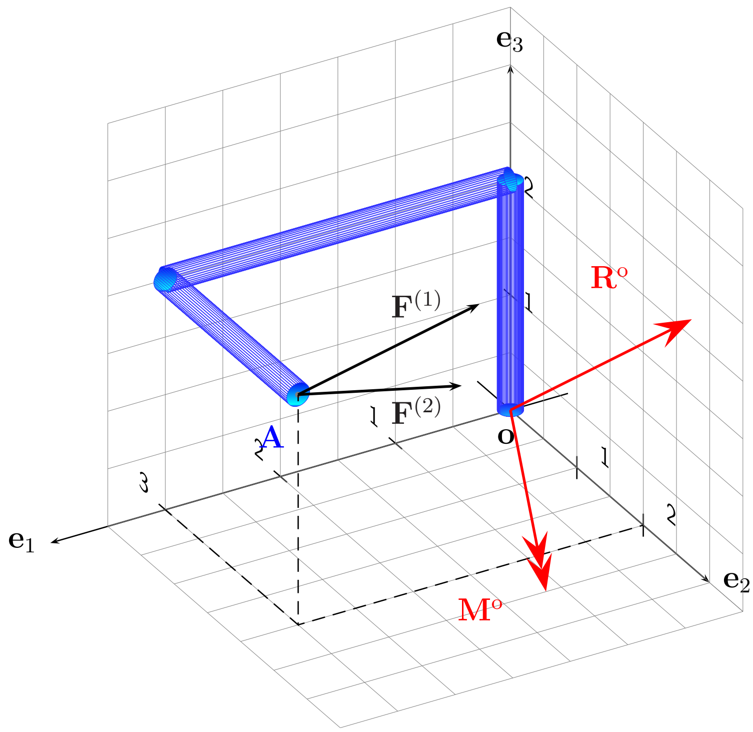


Figure 6: Pipe assembly: Free body diagram