16.001 Unified Engineering
Materials and Structures

Breguet Range Equation

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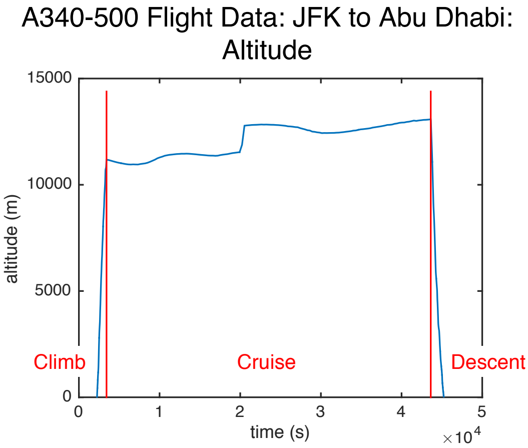
Outline

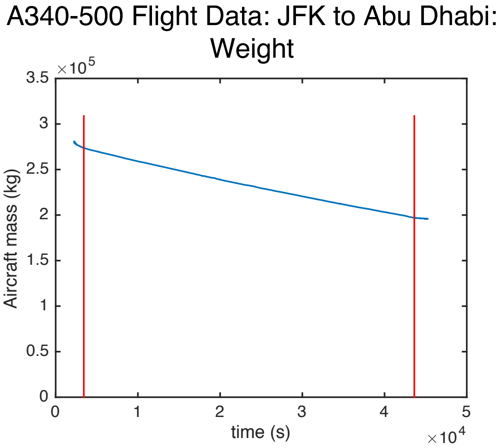
1. Estimation of powered aircraft maximum range: Breguet Range Equation
   - Learning Objectives
   - Some operational data
   - Historical note about the "Breguet" Range Equation
   - Derivation of the Breguet Range Equation
   - More practical and operational data

Learning Objectives

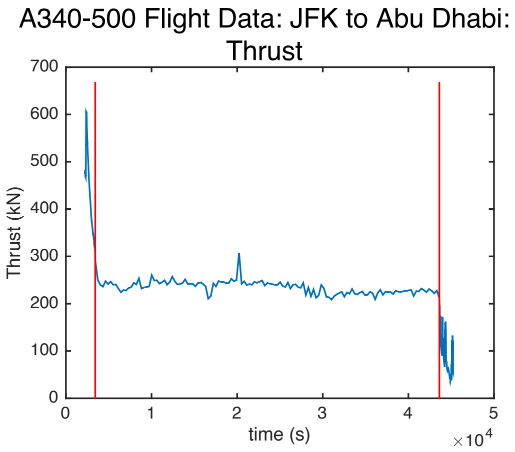
At the end of this lecture, you will be able to answer the following questions:

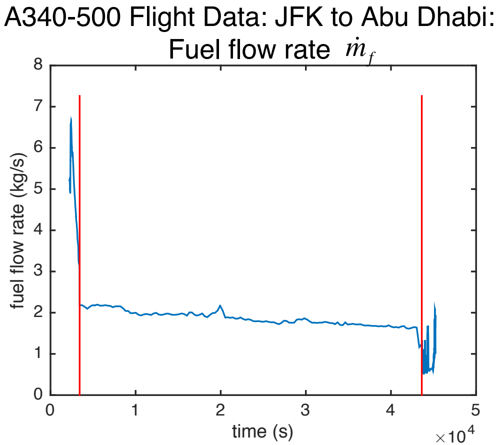
- How far can an airplane fly?
- How do the disciplines of structures & materials, aerodynamics and propulsion jointly set the performance of aircraft, and what are the important performance parameters?
- Estimate the performance of aircraft using empirical data and thus begin to develop intuition regarding important aerodynamic, structural and propulsion system performance parameters


A340-500 Flight Data: JFK to Abu Dhabi:
Altitude
15000
10000
5000
0
altitude (m)
0
1
2
3
4
5
time (s)
×10^4
Climb
Cruise
Descent


A340-500 Flight Data: JFK to Abu Dhabi:
Weight
×10^5
3.5
3
2.5
2
1.5
1
0.5
0
Aircraft mass (kg)
0
1
2
3
4
5
time (s)
×10^4

Some operational data III

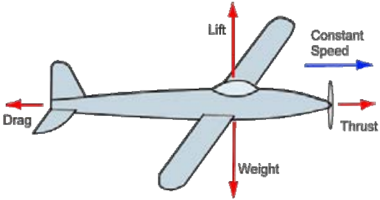



A340-500 Flight Data: JFK to Abu Dhabi:
Fuel flow rate $\dot{m}_f$
fuel flow rate (kg/s)
time (s)
$\times 10^4$
0
1
2
3
4
5
6
7
8
0
1
2
3
4
5

Historical note about the "Breguet" Range Equation

- According to the book Introduction to Flight, John Anderson, 2nd ed., McGraw-Hill, 1985, p.334), the earliest derivation of the range equation is found in a paper by J.G. Coffin, "A Study of Airplane Ranges and Useful Loads," NACA Report No. 69, 1919, with no reference to Breguet. It concludes that the reason for the association of the Range Equation with the name Breguet "...is historically obscure."

- Equilibrium or momentum conservation: During level flight, the aircraft travels at a constant ground speed V (m s^{-1}) and altitude, see figure:



Under these conditions, the propulsive force or thrust T (N) equals the aerodynamic drag force (D), and the aerodynamic lift force L (N) equals the total weight W (N):

$$\boxed{T = D}, \quad \boxed{L = W}, \quad \text{(N)} \tag{1}$$

You will study in Fluids in the spring that the drag and lift are related and that in level flight, the ratio of the two can be considered constant. This allows us to relate the vertical and horizontal equilibrium equations, and thus the thrust and the weight as follows:

$$W = L = \frac{L}{D} D = \frac{L}{D} T, \ \boxed{W = \frac{L}{D} T}, \ \text{(N)} \tag{2}$$

Note that L/D is a measure of the aerodynamic efficiency of the aircraft.

- Mass balance: At any point during flight, the total weight of the aircraft (W) is the addition of the weight of the structure (W_s), the weight of the payload (W_p), and the weight of the fuel (W_f):

$$\boxed{W = W_s + W_p + W_f}, \ \text{(N)} \tag{3}$$

The maximum total weight of the aircraft when the fuel tanks are full and under full payload is typically referred to with the acronym MTOW, or maximum take-off weight. Also, during flight, the total weight W changes as the fuel mass is expended. So the total aircraft weight as well as the fuel weight can be thought of as functions of time t or also distance traveled R. The change can be written mathematically as follows:

$$\boxed{\frac{dW}{dt} = \frac{dW_f}{dt} = -\dot{m}_f g}, \ (\mathrm{N\ s^{-1}}) \tag{4}$$

where $\dot{m}_f$ ($\mathrm{kg\ s^{-1}}$) is the fluid mass flow rate assumed constant in level flight, and $g \sim 9.81\ \mathrm{m\ s^{-2}}$ is the acceleration of gravity on earth (also assumed constant).

- Energy balance: The main consideration here is that the fuel energy is expended in producing the thrust which is necessary to counterbalance the drag. The propulsive power P_p is:

$$\boxed{P_p = T \times V}, \quad (\mathrm{N\ m\ s^{-1} = J\ s^{-1} = Watt}) \tag{5}$$

The power is provided by the combustion of the fuel, which has an amount of energy per unit mass h_f ($\mathrm{J\ kg^{-1}}$), and could in principle provide a fuel power P_f:

$$\boxed{P_f = \dot{m}_f \times h_f}, \quad (\cancel{\mathrm{kg}}\,\mathrm{s^{-1}} \times \mathrm{J}\,\cancel{\mathrm{kg^{-1}}} = \mathrm{Watt}) \tag{6}$$

However, and as you will study in detail in Thermodynamics and Propulsion, a number of losses occur in the process of converting the chemical energy available in the fuel to the final propulsive power. We refer to the fraction of the fuel power effectively contributing to propulsive power as the total efficiency η_0:

$$\boxed{\eta_0 = \frac{P_p}{P_f}} \tag{7}$$

and our energy balance principle can be written as:

$$\boxed{P_p = \eta_0 P_f}, \text{ or } \boxed{TV = \eta_0 \dot{m}_f h_f}, \text{ (Watt)} \tag{8}$$

We note that mass conservation Equation (4) gives us information on how the weight W changes over time t, not distance traveled R. However, distance traveled and time are related by: $\frac{dR}{dt} = V$, and we can use the chain rule to write:

$$\frac{dW}{dt} = \frac{dW}{dR}\frac{dR}{dt} = \frac{dW}{dR}V = \underbrace{-\dot{m}_f g}_{\text{from}(4)} \tag{9}$$

This expression can be combined with energy balance Equation (8), as follows:

from (8): $$\frac{\dot{m}_f}{V} = \frac{T}{\eta_0 h_f} \tag{10}$$

from (9): $$\frac{dW}{dR} = -g\frac{\dot{m}_f}{V} \tag{11}$$

combining: $$\frac{dW}{dR} = -g\frac{T}{\eta_0 h_f} \tag{12}$$

The final step is to now recognize that we haven't used our equilibrum (or momentum balance) Equation (2), which gives: $T = \frac{W}{\left(\frac{L}{D}\right)}$. Combining this with Equation (12), we get:

$$\boxed{\frac{dW}{dR} = -\frac{gW}{\eta_0 h_f \frac{L}{D}}} \tag{13}$$

This equation has the form:

$$W'(R) = aW(R), \qquad \text{where the constant coefficient : } a = -\frac{g}{\eta_0 h_f \frac{L}{D}} \tag{14}$$

It constitutes a first-order ordinary differential equation with constant coefficients and governs the evolution of the weight of the aircraft W as a function of distance traveled R. It can be easily integrated by noting that:

$$(\ln f(x))' = \frac{1}{f(x)} f'(x) \tag{15}$$

and therefore Equation (14) can be written as:

$$\frac{W'(R)}{W(R)} = (\ln W(R))' = a \tag{16}$$

$$\text{integrating:} \qquad \ln W(R) = aR + C \tag{17}$$

Now comes the important step of applying the initial condition, or known point in the solution $W(R)$. What we know, is that at the beginning of the flight (distance traveled $R = 0$), the weight of the aircraft is the total weight with full fuel tanks:

$$W(R = 0) = W_{\text{init}} = W_0 + W_{\text{fuel}} \tag{18}$$

Evaluating Equation (17) at the known solution point:

$$\ln W(R = 0) = a0 + C, \rightarrow C = \ln W_{\text{init}} \tag{19}$$

Replacing in Equation (17):

$$\ln W(R) - \ln W_{\text{init}} = aR, \ \rightarrow R(W) = \frac{1}{a} \ln \left(\frac{W}{W_{\text{init}}} \right) \tag{20}$$

Replacing the value of a from Equation (14):

$$R(W) = -\frac{\eta_0 h_f \frac{L}{D}}{g} \ln\left(\frac{W}{W_{\text{init}}}\right) \tag{21}$$

$$\boxed{R(W) = \frac{h_f}{g}\eta_0 \frac{L}{D} \ln\left(\frac{W_{\text{init}}}{W}\right)} \tag{22}$$

- the factor $\frac{h_f}{g}$ should define the dimension of the right hand side as all other factors are non-dimensional. Let's check this using SI units: h_f has units of energy per unit mass, or in SI: J kg^{-1} = kg m s^{-2}m$\cancel{\text{kg}^{-1}}$ = m^{2}s^{-2}, g has units of length per time squared, or in SI: m s^{-2}. Then, the ratio h_f/g in SI has units: $\frac{\text{m}^{\cancel{2}}\cancel{\text{s}^{-2}}}{\cancel{\text{m}}\cancel{\text{s}^{-2}}}$ = m. We conclude that this factor has dimensions of length and gives the dimension of the right hand side.
  Physically, this factor represents the efficiency of the fuel in terms of the energy density per unit mass. Clearly, a fuel with a higher value of h_f would increase the range, *ceteris paribus*. Typical values of h_f for jet fuel is around

40MJkg^{-1}. The equation also tells us that gravity affects the range in an inversely-proportional manner.

- The factor η_0 has already been discussed and represents the *propulsive efficiency* of the engine. Typical values for modern propulsion systems are around $0.2 - 0.4$.
- the factor $\frac{L}{D}$ is non-dimensional and represents the aerodynamic efficiency of the aircraft design. Typical values of $\frac{L}{D}$ in modern aircraft are around $15 - 20$.
- the factor inside the logarithm $\frac{W_{\mathbf{init}}}{W}$ represents the ratio of the sum of the structural W_s, payload W_p, and initial fuel W_f weights to the current total weight.

  The maximum range R_{max} for a given aircraft is obtained from Equation (22) when the initial weight $W_{\text{init}} = \text{MTOW}$, and all the fuel weight has been expended $W_f = 0$, in which case $W = W_0 = \text{OEW}$

$$\boxed{R_{max} = \frac{h_f}{g}\eta_0 \frac{L}{D} \ln\left(\frac{\text{MTOW}}{OEW}\right)} \qquad (23)$$

Clearly, $\frac{\text{MTOW}}{OEW}$ plays the role of a structural efficiency of the aircraft design, and calls for lighter and lighter aircraft where as much as possible of the weight is devoted to the fuel. Typical values of $\frac{\text{MTOW}}{OEW} \sim 2$.

THE BREGUET RANGE EQUATION

Or equivalently,

Speed of sound

Mach number
M = V/a

$$\text{Range} = \frac{a}{g} \frac{M \, L/D}{TSFC} \ln\left(\frac{W_{\text{init}}}{W_{\text{final}}}\right)$$

Thrust Specific Fuel Consumption
TSFC = mass flow rate of fuel per unit thrust

Warning: Watch units of TSFC which are typically kg/s/N or lbm/hr/lbf

FUEL ENERGY/UNIT MASS

Table 3 The heat of combustion or metabolic equivalent for various foodstuffs and fuels. The prices are based on a "snapshot" in 1994; large fluctuations may, of course, occur over time.

	MJ/kg[a]	$/kg	$/MJ	Comments
Prime beef	4.0	20	5	
Beef	4.0	8	2	
Whole milk	2.8	0.90	0.32	600 cal/quart
Honey	14	4	0.29	
Sugar	15	1	0.07	100 cal/ounce
Cheese	15	6	0.40	
Bacon	29	4	0.14	
Corn flakes	15	3.50	0.23	100 cal/ounce
Peanut butter	27	4	0.15	180 cal/ounce
Butter	32	4.50	0.14	
Vegetable oil	36	2	0.06	240 cal/ounce
Kerosene	42	0.40	0.010	0.82 kg/liter
Diesel oil	42	0.40	0.010	0.85 kg/liter
Gasoline	42	0.40	0.010	0.75 kg/liter
Natural gas	45	0.24	0.005	0.8 kg/m^3

a. megajoules per kilogram

(from <u>The Simple Science of Flight</u>, by H. Tennekes)

WEIGHT & GEOMETRY

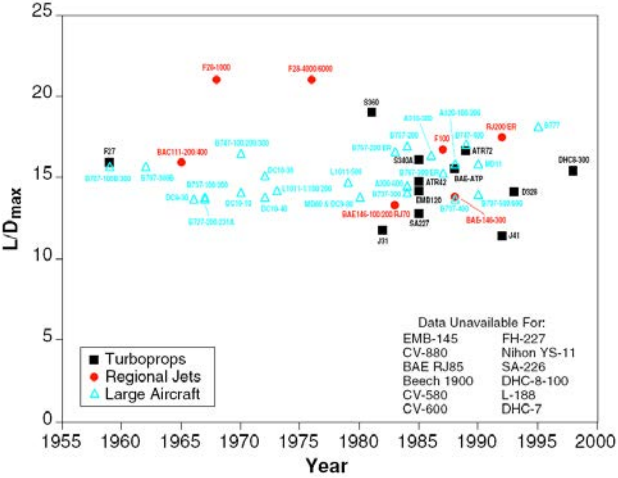
Table 5 Aspect ratio A and finesse F for various birds and airplanes. The values of A have been calculated from $A = b^2/S$; the values of F have been measured or estimated.

	W (N)	S (m^2)	b (m)	A	F
House sparrow	0.28	0.009	0.23	6	4
Swift	0.36	0.016	0.42	11	10
Common tern	1.2	0.056	0.83	12	12
Kestrel (sparrow hawk)	1.8	0.06	0.74	9	9
Carrion crow	5.5	0.12	0.78	5	5
Common buzzard	8.0	0.22	1.25	7	10
Peregrine falcon	8.1	0.13	1.06	9	10
Herring gull	12	0.21	1.43	10	11
Heron	14	0.36	1.73	8	9
White stork	34	0.50	2.00	8	10
Wandering albatross	85	0.62	3.40	19	20
Hang glider	1000	15	10	7	8
Parawing	1000	25	8	2.6	4
Powered parawing	1700	35	10	2.7	4
Ultralight (microlight)	2000	15	10	7	8
Sailplanes					
standard class	3500	10.5	15	21	40
open class	5500	16.3	25	38	60
Fokker F-50	19×10^4	70	29	12	16
Boeing 747	36×10^5	511	60	7	15

$$F = L/D$$

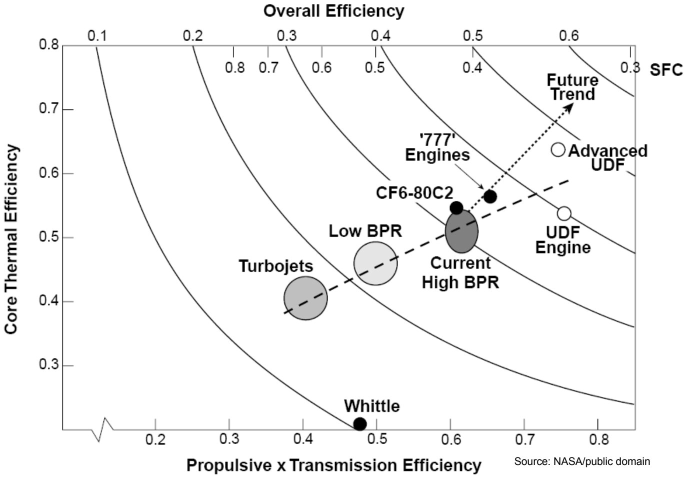
(from The Simple Science of Flight, by H. Tennekes)

AERODYNAMIC EFFICIENCY TRENDS



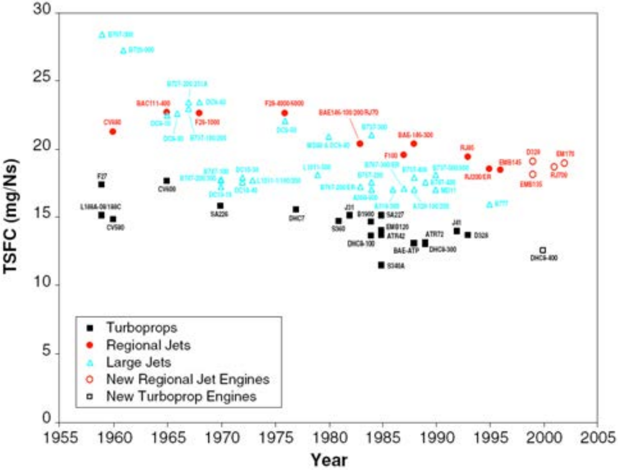
Babikian, Raffi, *The Historical Fuel Efficiency Characteristics of Regional Aircraft From Technological, Operational, and Cost Perspectives*, SM Thesis, Massachusetts Institute of Technology, June 2001

OVERALL PROPULSION SYSTEM EFFICIENCY



(After Koff, 1991)

ENGINE EFFICIENCY TRENDS
Turboprops, Regional Jets, Large Aircraft



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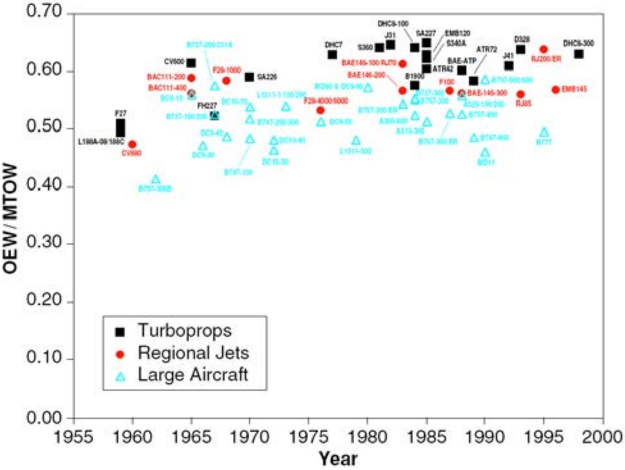
STRUCTURAL EFFICIENCY TRENDS
Turboprops, Regional Jets, Large Aircraft



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