16.001 Unified Engineering
Materials and Structures

Analysis of Planar Trusses

Reading assignments: Chap. 2 Connor's book, CDL 1.9

Instructor: Raúl Radovitzky
Teaching Assistants: Grégoire Chomette, Michelle Xu, and Daniel Pickard

Massachusetts Institute of Technology
Department of Aeronautics & Astronautics

Outline

1. Truss Analysis
   - Introduction to Truss Structures
   - Types of Truss Structures
   - Idealizations and hypotheses
   - Method of Joints
   - Method of Sections

Introduction to Truss Structures

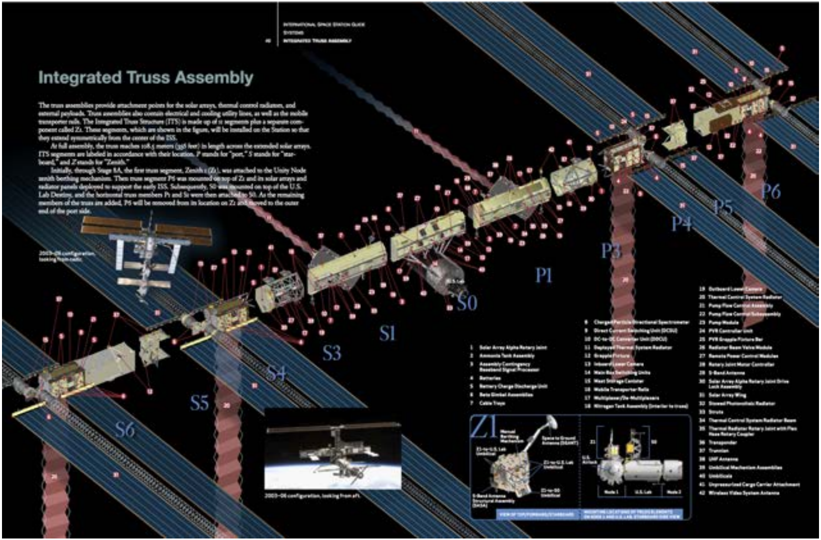
Definition

A truss is a structure formed by combining linear structural members in triangular patterns. The key structural characteristic of a truss is that the internal loads it carries are exclusively or primarily of an axial nature.
These structures are internally stable in the sense that the only change in shape under load is due to deformation of the truss elements.
Trusses have played a major role in structural design due to their unique structural efficiency characteristics.

ISS: integrated truss structure

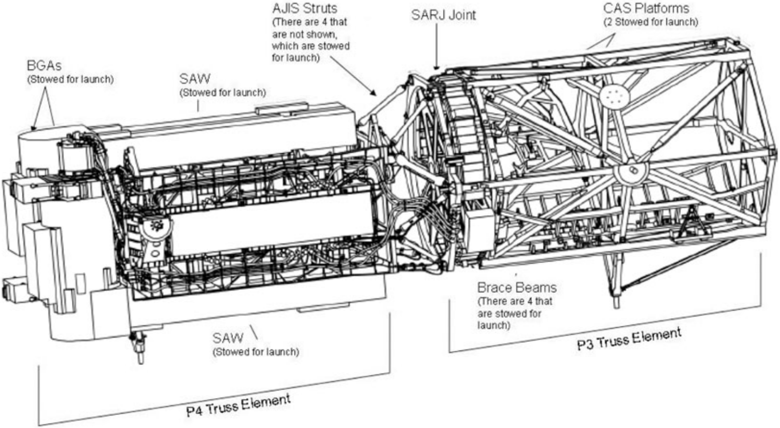
Source: NASA/public domain

ISS: integrated truss structure



Source: NASA/public domain

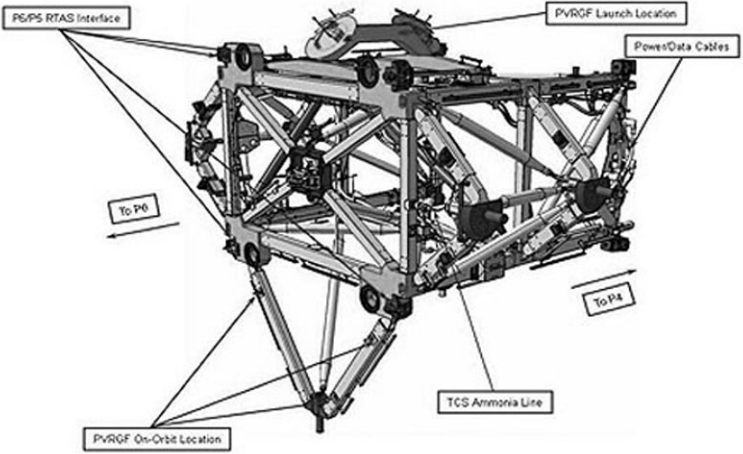
ISS: integrated truss structure



P3/P4 Truss design

Source: NASA/public domain

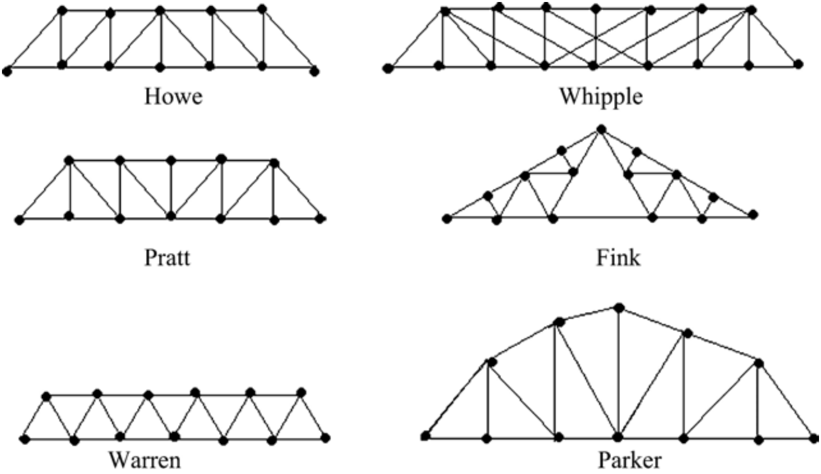
ISS: integrated truss structure

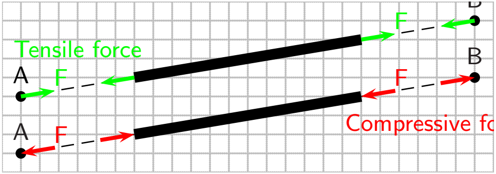


P5/S5 Truss design

Source: NASA/public domain

Types of Truss structures



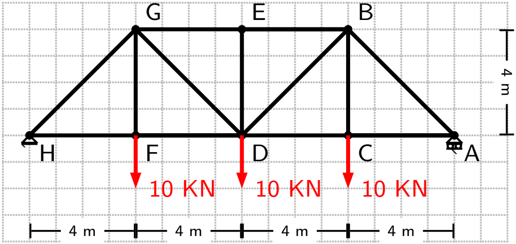


Idealizations and hypotheses

The analysis of trusses is based on the following idealizations which ensure that the internal forces in the truss members are purely axial (tension or compression):

- external actions (loads, supports, imposed displacements) are applied only at the nodes,
- truss members are connected with frictionless pins allowing their free relative rotation so that moments are not generated at the ends,
- the effect of the weight of the members on the internal loads is small compared to the applied loads,
- members are straight and their centroidal axis converge at a single point at joints.

Analysis procedure



Determination of reactions

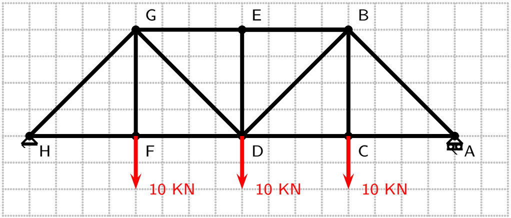
impose external equilibrium:

- FBD (replace supports with unknown reactions)
- use equilibrium equations
- if statically determinate, solve for reactions
- draw FBD with computed reactions and applied loads

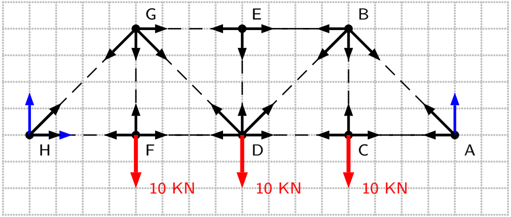
Determination of internal forces

We will explore two methods:

- Method of joints
- Method of sections

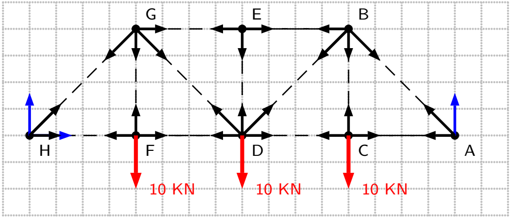


Each joint of a truss is subject to a concurrent force system. We can enforce equilibrium of each joint by:



Each joint of a truss is subject to a concurrent force system. We can enforce equilibrium of each joint by:

- isolating the joint, representing the member forces converging to the joint and external load applied at the joint (FBD)
- writing the two equations of force equilibrium which can be solved independently of other joints if there are at most two unknown member forces



Each joint of a truss is subject to a concurrent force system. We can enforce equilibrium of each joint by:

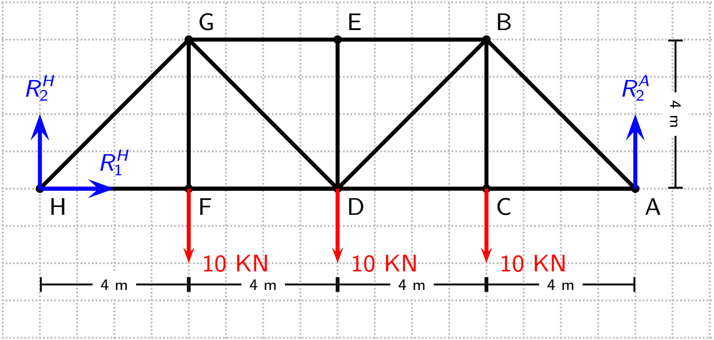
- isolating the joint, representing the member forces converging to the joint and external load applied at the joint (FBD)
- writing the two equations of force equilibrium which can be solved independently of other joints if there are at most two unknown member forces

If we do this for every joint in the structure, we obtain a system of linear equations whose unknowns are the internal member forces. If we can find a solution to this system, we have set each joint in equilibrium and therefore the whole structure is also in equilibrium.

Remarks:

- It is always convenient to work with components of the member forces in a common cartesian reference frame. The ratio of force components is equal to the ratio of the projected components of the member lengths (since the direction of the member force coincides with the direction of the member)
- The method of joints is most effective when one can progressively identify joints which have only two unknown member forces (each pair of joint equations is uncoupled).
- Identification of zero-load members: when a joint has only three members, two of which are colinear and there is no applied load at the joint

FBD - Reactions:



Equilibrium equations (Global):

$$\sum F_1 = \boxed{R_1^H = 0}$$

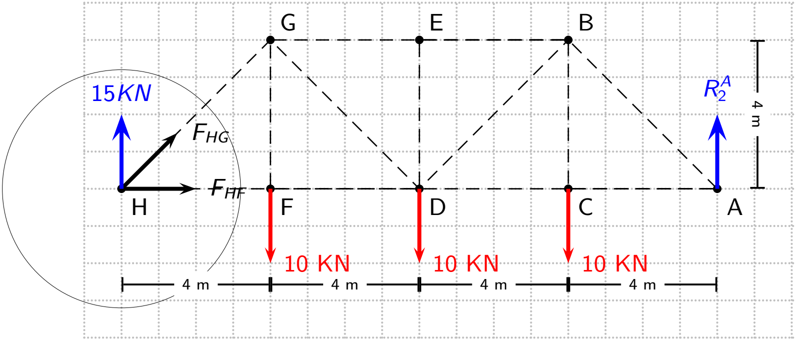
$$\sum F_2 = R_2^H + R_2^A - 30\text{KN} = 0$$

$$\sum M_3^H = 0$$

$$R_2^A \cdot 16m - 10KN \cdot (4m + 8m + 12m) = 0$$

$$\boxed{R_2^A = 15KN, \ R_2^H = 15KN}$$

Method of joints - FBD: Joint H



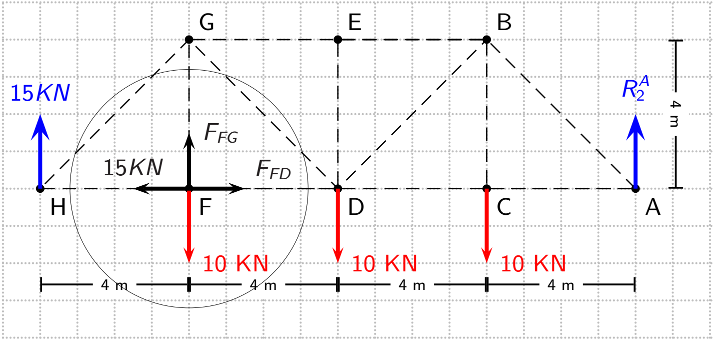
Joint Equilibrium equations:

$$\sum F_1 = F_{HF} + \cos(45^\circ) F_{HG} = 0$$

$$\sum F_2 = 15KN + \sin(45^\circ) F_{HG} = 0, \quad \boxed{F_{HG} = -\sqrt{2} \cdot 15KN}$$

$$\rightarrow \; F_{HF} = -\frac{\sqrt{2}}{2} \cdot (-\sqrt{2}) \cdot 15KN, \quad \boxed{F_{HF} = 15KN}$$

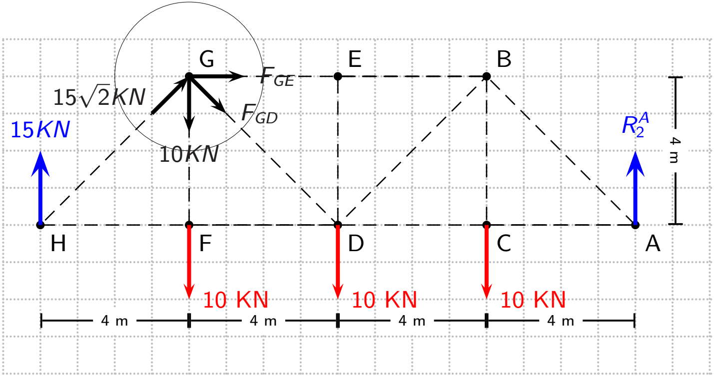
Method of joints - FBD: Joint F (can be done by inspection)



Joint Equilibrium equations:

$$\sum F_1 = F_{FD} - 15KN = 0, \rightarrow \boxed{F_{FD} = 15KN}$$

$$\sum F_2 = F_{FG} - 10KN = 0, \rightarrow \boxed{F_{FG} = 10KN}$$



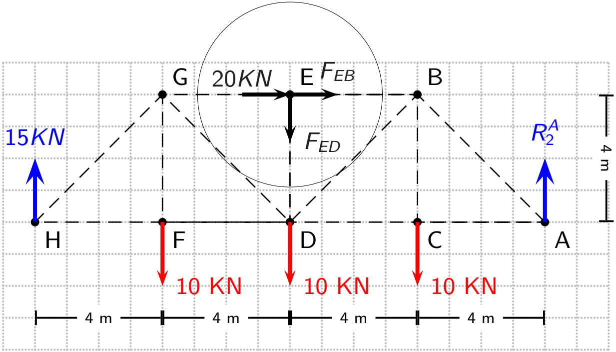
Joint Equilibrium equations:

$$\sum F_1 = F_{GE} + \frac{\sqrt{2}}{2}F_{GD} + \frac{\sqrt{2}}{2}(15\sqrt{2}KN) = 0,\ F_{GE} = -15KN - \frac{\sqrt{2}}{2}F_{GD}$$

$$\sum F_2 = -\frac{\sqrt{2}}{2}F_{GD} - 10KN + \frac{\sqrt{2}}{2}(15\sqrt{2}KN) = 0$$

$$\rightarrow \boxed{F_{GD} = 5\sqrt{2}KN, F_{GE} = -20KN}$$

Method of joints: Joint E (easily done by inspection)

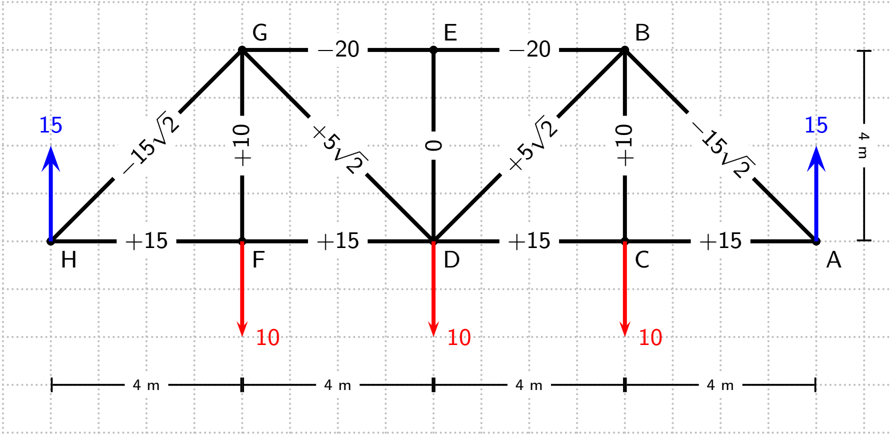


Joint Equilibrium equations:

$$\sum F_1 = F_{EB} + 20KN = 0, \rightarrow \boxed{F_{EB} = -20KN}$$

$$\sum F_2 = \boxed{F_{ED} = 0}$$

All other internal loads can be obtained by symmetry. Resulting schematic (all loads in KN):



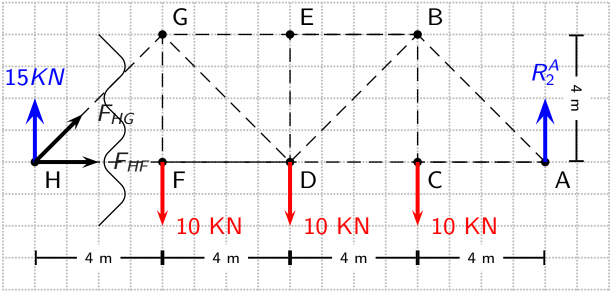
Method of Sections

- Convenient when one wants to determine force in a specific member without having to determine all of the other member forces previously.
- Provides insights on role of each member: allows to easily identify sense of member forces

Approach:

- pass a plane **through** truss making sure it cuts member whose force is to be determined.
- isolate (draw FBD of) either side of the structure
- use equilibrium equations to determine desired member force. Using equilibrium of moments is usually more convenient, as one can pick the center such that all but one unknown remains in the equation)

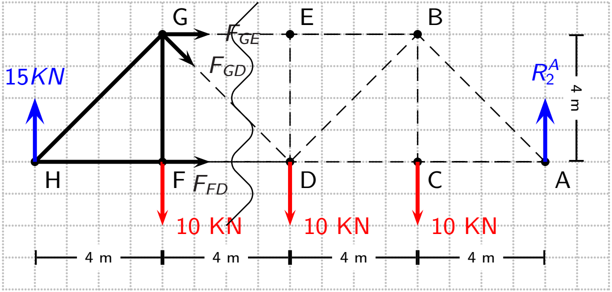
Reactions found the same way. FBD of "convenient" section of structure:



Take moments with respect to "convenient" point to isolate single unknown.
$\sum M^G = 0$ will give F_{HF}, $\sum M^F = 0$ will give F_{HG}:

$$\sum M^G = -15KN \cdot \cancel{4m} + F_{HF} \cdot \cancel{4m} = 0, \quad \boxed{F_{HF} = 15KN}$$

$$\sum M^F = -15KN \cdot \cancel{4m} - F_{HG} \cdot \frac{\sqrt{2}}{2}\cancel{4m} = 0, \quad \boxed{F_{HG} = -15\sqrt{2}KN}$$

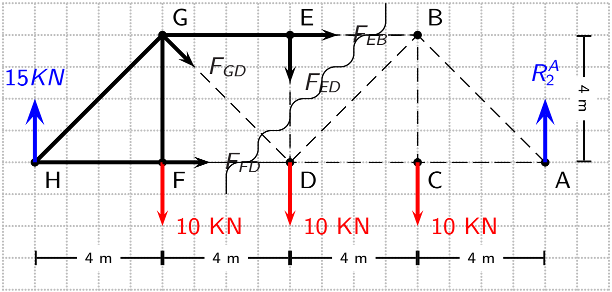


$\sum M^G = 0$ will give F_{FD}, $\sum M^D = 0$ will give F_{GE}, $\sum M^F = 0$ will give F_{GD}:

$$\sum M^G = -15KN \cdot \cancel{4m} + F_{FD} \cdot \cancel{4m} = 0, \quad \boxed{F_{FD} = 15KN}$$

$$\sum M^D = -15KN \cdot 2 \cdot \cancel{4m} + 10KN \cdot \cancel{4m} - F_{GE} \cdot \cancel{4m} = 0, \quad \boxed{F_{GE} = -20KN}$$

$$\sum M^F = -15KN \cdot \cancel{4m} - \underbrace{F_{GE}}_{-20KN} \cdot \cancel{4m} - F_{GD} \cdot \frac{\sqrt{2}}{2} \cancel{4m} = 0, \quad \boxed{F_{GD} = 5\sqrt{2}KN}$$



$\sum M^G = 0$ will give F_{ED}:

$$\sum M^G = -15KN \cdot \cancel{4m} + \underbrace{F_{FD}}_{15KN} \cdot \cancel{4m} - F_{ED}\cancel{4m} = 0, \quad \boxed{F_{ED} = 0KN}$$

or also trivially from node E.