

- Problem 1** (Morris contagion model). (a) (Exercise 9.16 in Jackson) Consider a network (N, g) where each node take action $a \in \{0, 1\}$, and action 1 is the optimal action for a node if and only if a fraction of at least q of his or her neighbors take action 1. Show that a sufficient condition for never having a contagion from any group of m nodes is to have at least $m + 1$ disjoint sets of nodes that are each more than $(1 - q)$ cohesive.
- (b) Consider a variant of the Morris contagion model where in period $t = 0$ some nodes play $a = 0$ and others play $a = 1$ (arbitrarily), and subsequently in each period t each node i plays $a = 1$ if and only if at least $q = 0.5$ of its neighbors played $a = 1$ in period $t - 1$. (The difference from the model in lecture is that now nodes can switch from $a = 1$ to $a = 0$ in addition to switching from $a = 0$ to $a = 1$.) Give an example where this process cycles forever.

Problem 2. In each part of this question, we verbally describe a classic multi-agent decision problem. Formally express each one as a normal form game, and find all (pure and mixed) Nash equilibria.

- (a) *Partnership*: Two partners in a firm each decide whether to work or rest. Each partner earns \$100 of profit for the firm if she works, regardless of what the other partner does. All profits earned are divided equally between the two partners. Each partner also derives a private benefit worth \$75 to her from resting. This private benefit cannot be shared with the other partner.
- (b) *Stag Hunt*: Two hunters can each hunt stag or hare. If both hunt stag, they catch a stag and get 100 pounds of meat each. If a hunter hunts hare, she catches a hare and get 10 pounds of meat, regardless of what the other hunter does. If a hunter hunts stag while the other hunts hare, the hunter hunting stag catches nothing.
- (c) *Chicken*: Two drivers approach each other on a narrow road. Each can either continue or swerve. If one continues and the other swerves, the driver who continues gets a payoff of 1 for appearing brave, and the driver who swerves gets a payoff of 0 for appearing cowardly. If both swerve, both get a payoff of 0. If both continue, they collide and both get a payoff of -10.
- (d) *Rock-Paper-Scissors*: Two players each decide whether to throw Rock, Paper, or Scissors. Rock beats Scissors, Scissors beats Paper, and Paper beats Rock. The winner gets \$1 from the loser. No money changes hands in case of a tie.
- (e) *Modified Rock-Paper-Scissors*: Same as above, but now the amount of money won/lost is not always \$1. Instead:

Rock *crushes* Scissors: winner gets \$10 from loser when Rock beats Scissors.

Scissors *cut* Paper: winner gets \$5 from loser when Scissors beats Paper.

Paper *covers* Rock: winner gets \$2 from loser when Paper beats Rock.

Ties are treated as in standard Rock-Paper-Scissors.

Problem 3. This problem investigates “Downsian political competition” (introduced by Anthony Downs in his book, *An Economic Theory of Democracy*), a major paradigm in political science.

Consider a population of voters uniformly distributed along an ideological spectrum from left ($x = 0$) to right ($x = 1$). Each of the candidates for a single office simultaneously chooses a campaign platform (i.e., a point on the line between $x = 0$ and $x = 1$). The voters observe the candidates’ choices, and then each voter votes for the candidate whose platform is closest to the voter’s position in the spectrum.

For example, if there are two candidates and they choose platforms $x_1 = .3$ and $x_2 = .6$, then all voters to the left of $x = .45$ vote for candidate 1 and all those to the right of $x = .45$ vote for candidate 2, so candidate 2 wins with 55% of the vote. Assume that any candidates who choose the same platform equally split the votes cast for that platform, and that ties among the leading vote-getters are resolved by coin flips.

- (a) Suppose there are two candidates, and that the candidates solely try to maximize their probability of getting elected (this is called *office-motivated candidates* in political science). Solve for the pure-strategy Nash equilibrium, and prove that it is unique. (*Extra credit: prove that there is no mixed-strategy NE.*)
- (b) Suppose there are two candidates, and that now the candidates care solely about the winning platform and not about who wins (this is called *policy-motivated candidates*). Specifically, Candidate 1 is a left-winger, Bernard: if the winning candidate chose platform x , Bernard’s payoff is $1 - x$. Similarly, Candidate 2 is a right-winger, Don: if the winning candidate chose platform x , Don’s payoff is x . Note that except for the payoffs the game is exactly the same as in part (a). Solve for the unique pure-strategy Nash equilibrium.
- (c) Suppose candidates are office-motivated as in part (a), but assume there are now three candidates. Find one pure-strategy NE.

Problem 4. Consider the example of inefficient routing with non-linear latency from Lecture 12; there are two links from origin to destination, with $l_1(x) = x^k$ and $l_2(x) = 1$, where k is a positive number.

- (a) Find the socially optimal routing and the equilibrium routing, and calculate the price of anarchy/stability. (Your answers will depend on k .)
- (b) Now suppose that the mass of traffic that must be routed from origin to destination is 2 rather than 1 (while the functions $l_1(\cdot)$ and $l_2(\cdot)$ are unchanged). Again, find the socially optimal routing and the equilibrium routing, and calculate the price of anarchy/stability.
- (c) Verify that, for any value of k , the total *equilibrium* delay with traffic 1 is less than the total *socially optimal* delay with traffic 2.

In fact, this is a general phenomenon: for any network, the socially optimal routing is worse than the equilibrium routing with half as much traffic. (You do not have to prove this.) Can you think of any implications of this fact for how society can best control traffic?

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