

Problem 1 (Centrality Measures). Consider each of the following two networks

(a) The directed network represented by

$$g = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(b) A undirected star network with $N + 1$ nodes

Answer the following questions:

1. Draw each of the networks.
2. Calculate, for each case, the vectors of eigenvector centrality and PageRank for $\alpha = 0.25$ and $\alpha = 0.5$.
3. Comment on how PageRank changes as you increase α . Is this consistent with your intuition that increasing α allocates more centrality to “indirectly” important nodes?
4. In each network, try to compute Katz-Bonacich centrality for $\alpha = 0.5$. For what values of N in the star network (b) is Katz-Bonacich centrality ($\alpha = 0.5$) well-defined?

Hint: if you do not want to invert matrices, try writing out the series expansion $(I - \alpha A)^{-1} = I + \alpha A + \alpha^2 A^2 + \dots$ in each case.

Bonus: use a computer to replicate this exercise for the following matrix:

$$g = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Experiment with different values of α for PageRank and Katz-Bonacich centrality. Can you find the upper bound for α such that Katz-Bonacich Centrality remains well-defined? How does this relate to the leading (largest-norm) eigenvalue of g' ?

Problem 2 (Production Networks, in Practice). Pick **one** of the following three articles about measuring how economic shocks affect production networks:

1. Barrot and Sauvagnat, “Input Specificity and the Propagation of Idiosyncratic Shocks in Production Networks”
2. Boehm and Pandalai-Nayar, “Input Linkages and the Transmission of Shocks: Firm-Level Evidence from the 2011 Tohoku Earthquake”
3. Carvalho, Nirei, Saito, and Tahbaz-Salehi, “Supply Chain Disruptions: Evidence from the Great Japanese Earthquake”

You can find links to the articles on Stellar. Read your selected article at a level of detail sufficient for you to answer the following questions regarding the paper’s empirical approach. You should not try to read every section of the article you choose, as parts of these articles involved advanced economic analysis that is beyond our scope. A good tip would be to focus on the introduction, data description, and “main results” sections.

1. Any empirical study of networks needs to take a stand on how “big” a node is. What definition does the study use? Can you imagine a finer or coarser definition? Comment briefly on whether you think the study’s definition is the “right” one based on the study’s goals. [Concise paragraph]
2. Similarly, studies need to take a stand on what edges are. How does your study define a supply-chain link? Does this seem like a reasonable definition to you? [Concise paragraph]
3. Summarize what *you* consider the paper’s most important empirical result—use numbers and be specific about the units. [One or two sentences]
4. Elon Musk, CEO of Tesla and SpaceX, is [worried about supply chain risks](#). He has requested an executive summary of the study you read along with some suggestions for improving the “resilience” of his companies in future crises. These suggestions can be acted upon by a team of operations engineers, who have detailed data on Tesla’s and SpaceX’s suppliers. What do you say? [Slightly longer paragraph]

Hint: Here are definitions for some “jargon” that may come up:

- *Endogenous*: having an “internal” cause, or not being an “external” shock

- *Exogenous*: opposite of endogenous
- *Elasticity*: percentage change in response to percentage change
- *Elasticity of substitution*: a measure of how easy it is to switch between two different inputs to produce the same output (higher = easier)
- *Compustat*: a dataset of public firms' financials, like what you might read in the newspaper (Facebook's Q4 sales were \$X and its profits were \$Y...)
- *Establishment*: a single physical location where economic production occurs. e.g., one Starbucks is an establishment, whereas all of Starbucks is a *firm*.

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14.15 / 6.207 Networks
Spring 2022

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