These problems are for ***practice only*** *and are not to be turned in. You are responsible for this material for the final exam. You should be able to do the first two problems now and they should be able to answer the remaining two problems after the final week's lectures.*

Problem 1. Alice and Bob are trying to meet for lunch. They can each go to the Cafe or the Diner. Alice's office is near the Cafe, so she knows the exact length of time w it would take to wait in line at the cafe. Bob's office is far from the Cafe, so all he knows is that w is distributed $U[0,2]$. All else equal, Alice would be equally happy eating at the Cafe and the Diner, but Bob prefers eating at the Cafe by an amount b that varies from day to day: assume that Bob knows the exactly value of b, while Alice knows only that b is distributed $U[0,3]$, independently of w. In addition, Alice and Bob get a benefit of 1 from having lunch together. Summarizing, with Alice as player 1 and Bob as player 2 the payoff matrix is

	C	D
C	$1-w, 1-w+b$	$-w, 0$
D	$0, -w+b$	$1, 1$

(a) Formally model this situation as an incomplete information game.

(b) Find a BNE, and prove that it is unique. How often do Alice and Bob have lunch together?

Problem 2. Consider a seller who must sell a single good. There are two potential buyers, each with a valuation for the good that is drawn independently and uniformly from the interval $[0, 1]$. The seller will offer the good using a second-price sealed-bid auction, but he can set a "reserve price" of $r \geq 0$ that modifies the rules of the auction as follows: If both bids are below r then neither bidder obtains the good and it is destroyed. If both bids are at or above r then the regular auction rules prevail. If only one bid is at or above r then that bidder obtains the good and pays r to the seller.

(a) Compute the seller's expected revenue as a function of r.

(b) What is the optimal value of r for the seller?

(c) Intuitively, why does the seller benefit from setting a non-zero reserve price?

Problem 3 (Herding model)**.** Jackson, Problem 8.6, pg. 254.

Problem 4 (DeGroot learning)**.** Consider the DeGroot learning model with N agents with initial belief vector $x(0) = (x_1(0), \dots, x_N(0))$ and an $N \times N$, non-negative, row stochastic matrix T such that, for every period t, we have

$$x(t) = Tx(t-1).$$

(a) Suppose that $N = 3$ and

$$T = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{1}{3} & \frac{2}{3} \end{pmatrix}.$$

What properties of this matrix guarantee that, for any initial belief vector $x(0)$, the limit belief $x^* = \lim_{t\to\infty} x(t)$ is well-defined? Compute x^* as a function of $x(0)$.

(b) Suppose that $N = 3$ and

$$T = \begin{pmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

As a function of $x(0)$, compute $x(t)$ for every $t \geq 1$. What is going on?

[Hint: First compute T^2 and T^3, then compute $x(1)$, $x(2)$, and $x(3)$. You should notice a pattern.]

(c) Prove that, for any N, if there exists an agent i such that $T_{ii} = 1$ and $T_{ji} > 0$ for all $j \neq i$, then $x_j^* \equiv \lim_{t\to\infty} x_j(t)$ is well-defined and equal to $x_i(0)$ for all $j \neq i$.

[Hint: Let $\Delta(t) = \max_{j\in N} |x_i(t) - x_j(t)|$ and let $\underline{T} = \min_{j\neq i} T_{ji}$. Prove that $\Delta(t+1) \leq (1 - \underline{T})\Delta(t)$ for all t. Show that this implies that each $x_j(t)$ must converge to $x_i(0)$ as $t \to \infty$.]