# Problem Set 4

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## 1 Walrasian Equilibrium with Production

There is an economy with 2 goods and $I$ consumers. Each consumer $i$ has the following utility function:

$$u^i(x_1^i, x_2^i) = x_1^i + \log x_i^2$$

Each consumer starts with an endowment of 4 units of good 1 and none of good 2. Good 2 can be produced from good 1 using the following production function:

$$y_2 = \sqrt{z_1}$$

where $z_1$ is the amount of good 1 used as an input and $y_2$ the output of good 2 produced. There is a single firm that produces good 2 from good 1 and each consumer owns an equal share of this firm, so they each get an equal share of profits.
The price of good 1 is normalized to 1. Let the price of good 2 be $p$. profits.

a) Write down the conditions that a price $p$ and an allocation $(\{x_1^i\}_{i=1}^I, \{x_2^i\}_{i=1}^I, y_2, z_1)$ must satisfy to be a Walrasian equilibrium.

b) Solve the firm's profit maximization problem for an arbitrary $p$. What is its profit function, $\pi(p)$?

c) Solve the consumer's utility maximization problem for an arbitrary $p$.

d) Using your answers to b) and c) and market clearing, find the Walrasian equilibrium.

e) Calculate each consumer's utility in equilibrium. How does it depend on $I$? Can you give any intuition for this result?

## 2 Trade and the 2x2x2 Model

Suppose there are two goods $x, y$, two factors $K, L$, and countries $A, B$ have the same preferences and technology. Preferences follow the function

$$U_i(x_i, y_i) = 2\sqrt{x_i} + 2\sqrt{y_i}$$

and production technology is Leontief with good $x$ following the production function

$$x = f(K_x, L_x) = \min\{2K_x, L_x\}$$

and good $y$ following the production function

$$y = g(K_y, L_y) = \min\{K_y, L_y\}$$

Country $A$ has endowments $\omega_A = (\overline{K_A}, \overline{L_A}) = (20, 30)$, and country $B$ has endowments $\omega_A = (\overline{K_A}, \overline{L_A}) = (35, 50)$. The prices paid for capital and labor are $r$ and $w$ respectively.

a) Suppose the world is in autarky. We will go through the steps to solve for equilibrium prices $(p_x^c, p_y^c, r^c, w^c)$, consumption, and factor allocations in the two countries.

   i) We will find the equilibrium conditions for the production side first. What are the unit-cost functions for production of the two goods?

   ii) Which good is more capital-intensive?

   iii) Notice that the production function is CRS and therefore profits are zero. What are the equations determining prices $p_x, p_y$ as a function of $r$ and $w$?

   iv) From firm optimization and market clearing for the factor endowments for each country, calculate the factor allocations $K_x^c, L_x^c, K_y^c, L_y^c$ for each country $c \in \{A, B\}$. Calculate total production for goods $x^c, y^c$.

   v) Now we move to the consumer side. From utility maximization (assume an interior solution), what must the price-ratio equal?

   vi) We find the general equilibrium by putting the production and consumer conditions together. Using the zero-profit equations and the utility maximization condition, solve for equilibrium prices $(p_x^c, p_y^c, r^c, w^c)$.
   *Hint*: Use Walras' Law and normalize wage $w^c = 1$.

b) Suppose now there is free trade. We will go through the steps to solve for equilibrium prices, consumption, factor allocations, and net exports from $A$ to $B$.

   i) With free trade and the same technology and preferences in both countries, what does this tell us about good and factor prices in country $A$ compared to country $B$?

   ii) Using your answer in (i), solve for good and factor prices.

   iii) Do factor allocations and production change with free trade? How about consumption?

   iv) Calculate net exports from $A$ to $B$ for goods $x$ and $y$. *Hint:* calculate total income for each country, then solve for consumption using the good prices.

c) Interpret your results from c) in terms of the Heckscher-Ohlin theorem.

# 14.04: Intermediate Microeconomic Theory

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