[SQUEAKING]

[RUSTLING]

[CLICKING]

ROBERT TOWNSEND: OK, so let me say a couple of words about where we are in the reading list. We're going to start today. We're done with the segment on contracts and mechanism design, theory and applications.

And we start today, Walrasian General Equilibrium, which you've seen a bit before already. But we'll review and give the proper definitions for the first 20% of the lecture. And then the rest of the lecture is devoted to an important application of General Equilibrium theory. And in particular, it has to do with trade.

So this lecture today is the theory, with some really startling and interesting propositions. And then with trade on our minds, we'll go back to the Thai data, and take a look at trade flows and financial flows that have changed over time as the country became more liberalized, or more open internally.

And then we're going to go to a parallel lecture on the US that's going to look at trade flows and potentially financial flows across states in the US, where we'll be analyzing the impact of tariffs, and/or China shocks-- China imports.

So these, dare I say it, it may or may not be on your mind. If it's on your mind already, the motivation here is a US policy decision which is very much in the news, and in some respects, a big part of the upcoming election. So I don't know if the timing of these lectures are good or bad. But anyway, we'll be talking about US trade policy within two lectures.

So that's what's coming up. And today's lecture has got two start readings, the Kreps book on section 6.1, and this Mas-Colell, Whinston, and Green. I rarely assign this book. But on the other hand, the lecture is based very closely on that chapter. So I went ahead and just listed it, MWG chapter 15 D, although it's quite a famous model. We've been using MWG in the theorems and so on. But I've never really featured it so heavily.

In terms of the overall-- just to see it in one spot, again, we're going to do Walrasian equilibrium, and then two applications of trade in Thailand and the US. That's actually going to complete-- lecture 15 completes the current segment of lectures before the second exam. So problem set 4, which was released last-- the last thing I want to do before getting to the lecture is to review a bit of what we did last time.

I deliberately didn't pick out very many questions here. Allow risk aversion on the part of the borrower. There are several advantages to writing down programs in lotteries. List two of them and explain. So let's see if I can get some-- Charles, do you want to take a crack at that?

AUDIENCE: I think I can guess one, definitely at least one of them, which, if I remember correctly, is that it allows you to make an incentive. You may select between people with high income and low income, if you don't know the income already.

So like, if you're risk averse in the correct manner, then if you have a lottery, then someone with low income might prefer a lottery with something like-- you might prefer the different risk return trade off at different incomes.

There's a dislike between various things, and help with some of the constraints. The other one, I don't remember the exact details. I haven't quite got them yet.

ROBERT TOWNSEND: The first one is good.

AUDIENCE: I'm going to guess something like, maybe making people not as willing to borrow or something like that. But I'm not exactly sure.

ROBERT TOWNSEND: Let's see. Kerry, can you get one more of them?

AUDIENCE: Is it, like, incentive constraints could be non-convex. So it might be hard to solve.

ROBERT TOWNSEND: Yeah, good. So there's various sources of non-convexity. So those incentive constraints, when written without the lotteries, like first order conditions of the borrower's choice of effort. Those things can lead to non-convexity. So by doing it in lotteries, we get that problem solved.

And also, if there is some indivisibilities, that would be solved. There are two indivisibilities in this problem. One has to do with occupation choice. You're either a wage earner or an entrepreneur. That's a binary choice. So the lotteries kind of smooth that over because you can effectively choose the probability.

And secondly, there could be individual abilities in the discreteness of the capital grid. Like, you're going to capitalize a high amount or a low amount. And again, lotteries help with that indivisibility.

OK, so the next one is really a summary of what we did. But let me read it out. Sketch out how one can take the model with the lotteries to the data on occupation choice, and estimate parameters. So can anyone attempt a summary of what we did last time on that dimension?

Pablo?

AUDIENCE: Sure, I can take a stab at this. So I think the way that we did it was we-- given these different parameters that we had, of ability, education, and wealth, we solved for this optimal contract, theoretically. And then we made a likelihood function to-- given these solutions, to see which one of these is like going to be like the best, essentially.

ROBERT TOWNSEND: Do you remember how we did, quote, "the best?" The best in terms of what?

AUDIENCE: Just kind of, like, the likelihood function, trying to maximize that.

ROBERT TOWNSEND: Yeah, OK. Yeah, so the likelihood function tells us the probability of what we are seeing in the data if the model were true. So we're choosing parameters to make that probability as high as possible. OK, that's good.

Any other questions about that lecture? All the other questions at the beginning here were basically, write it down the way we did it in class, et cetera, et cetera. So it's not exactly conducive to asking you these questions in class. But it is important to be reviewing these review questions.

All right, so then we come to the lecture today. This is Walrasian equilibria with an application to trade. And we're going to consider trade, for the most part, in a small open economy. Although we'll say a little bit about putting the world pieces together at the very end. So here's the notation, which is largely a review because you have seen almost all of it before.

When we write down an economy, we mean the commodity space, in this case, capital L, a finite number of commodities. I is the number of consumers. J is the number of firms-- finite for each. Household or consumer, I, has a consumption set. And preferences over bundles in the consumption set, as for, example, represented by a utility function.

Each firm, J, has a technology production possibility set in the same L dimensional space. And the economy has exogenously given endowments, according to this L dimensional endowment vector, with the bars being the aggregates.

So that's the economy. Now, in, say, a private ownership economy, we have to specify, in addition, who owns what. So we imagine that consumer I has an endowment vector, L dimensional, subscript I, for consumer, I. And it is the sum of these endowments over all the household that adds up to the omega bar L dimensional endowment vector on the previous slide. Aggregates naturally come from the individual's ownership.

Each consumer, in addition to having endowments, has some share of the profits of firms. So theta, IJ, being the share that household, I, has, of J's profits. So these are meant to be a division of the profit. So the sum of the shares over households, I, for a given firm, J, add up to 1. Everything gets distributed. And household I is getting theta IJ of pi J, for firm, J. Now, pi J is obviously an endogenous object.

It could be 0 if there were constant returns to scale. It could be positive if there are diminishing returns to scale. And also, when we did risk and return in village economies, we kind of naturally had an interesting and relevant special case that household I gets all the profits from the activities that it undertakes.

In that case, there was a household, I, and a firm, I. It was the same entity. And household I was getting theta IJ equal to 1 of its own profits, as a special case.

But general equilibrium theory usually allows something more general. And you could think of this literally as a market economy, where the shares are equity shares being traded. And all of the dividends are claimed by the ownership of shares. OK, so a private ownership economy has a summary of the consumption, set preferences, endowments of consumer I, the production sets of all the house firms, J, and the shares, theta IJ.

Now we get to the big definition here. In this L dimensional commodity space, a price vector, p, is the price of all the commodities, inputs, and outputs, and everything. And a Walrasian equilibrium, or competitive equilibrium for short, for this private ownership economy, is a specification of allocations with stars on them, x and y star, for consumers and firms-- and a price vector star here because it's a special price.

Allocations and prices have the special properties. Each firm, J, has y star J as the maximum of the firm's problem, which is to maximize profits. Hence, at y star, the valuation of outputs less inputs is weakly greater than it would be for any other choice of production vector.

Each consumer, i, is maximizing relative to the preference ordering on bundles in the consumption set, but also subject to the budget constraint, that expenditures at price p star, for any xi that could potentially be chosen-- cannot exceed the right hand side, which is the private ownership part. So omega i, being the endowments of consumer, i, evaluated at price p star, that dot is inner product.

And again, household i's claim on the profits of firm, J. And the final property is that the allocation has to be feasible. And you can just read this as demand cannot exceed supply, where supply includes both the aggregated endowment and production. So we worked a lot with feasibility before. We did that when we defined Pareto optimality.

The difference here in general equilibrium or Walrasian equilibrium is we've added this price vector explicitly, and rewritten consumer maximization problem, on which we've had several lectures. And firms-- sorry, I reversed the order-- firms max and consumers max problems. So everything's in one spot.

OK, so a classic example is the Edgeworth Box. We previously defined Pareto optimal allocations in this box, as characterized by tangencies of indifference curves. Here we add one more ingredient, that if they start with this endowment, and we draw a price line through the endowment, then each consumer is maximizing utility, subject to its budget.

So consumer 1 here has the normal-looking budget line. And coming from the endowment and the tangency at x star would be the maximum. You may remember consumer 2 up here on the northeast is moving to the southwest in terms of increased utility, and is also maximizing utility, which is achieved by a tangency on that consumer's budget line.

So this line here is doing double duty. It's a budget line for each household. But the perspective is at or below consumer 1, at or above the line for consumer 2. And here you're seeing the star allocations, which are a competitive equilibrium. Everything in the box adds up to the aggregates by construction. There is no production. They're all maximizing utility.

And the price vector is characterized by the slope of this budget line. Now, we could go on with a lot more interesting, and some nonstandard pictures, of the Edgeworth Box. But we're going to save a lot of that for another lecture. Here, I defined Walrasian equilibrium. I wanted to find something similar but not identical. It's a price equilibrium with transfers.

So here, the idea is that the right hand side is a wealth assignment. Household, i, has wealth, w sub i. So the rest is similar. We're going to have an allocation with stars-- a price vector, p, which should have had stars on it because it's a special one-- with the property, and an assignment of these wealth levels, such that firms maximize profits. That's the same as before.

Households maximize utility. But the right hand side of the budget is this wealth object. And again, when we did partial equilibrium consumer problem, we talked about prices and wealth. We had income expansion paths as wealth varied. So this formulation of the problem is consistent with that. Although in partial equilibrium, we didn't ask where that wealth was coming from.

An allocation is feasible. Again, demand cannot be greater than the two sources of supply. Now the wealth assignments have to be feasible. And here's where we can say something about where wealth is coming from, namely, the sum of the wealths, whatever they are, have to equal the valuation of the aggregated endowment vector plus the profits of all the firms.

So somehow, this stuff on the right hand side gets distributed across the households, each household, i, getting wealth, wi. And when we add it up, therefore, it sums up to the right hand side. Now note that at the bottom here, a particular wealth assignment that works is what we had on the previous slide, namely, give household, i, the valuation of its endowment, plus its share of the profits of the firms that it owns.

So in that sense, a price equilibrium with transfers is more general, but includes the valuation equilibrium. That said, the word transfers may catch your eye. And the idea here is that a Walrasian equilibrium with ownership specifies one level of wealth. But we could, in principle, redistribute that wealth by the government with lump sum taxes and transfers, and then look for a new priced equilibrium.

And we will-- I think it's on the next slide, yeah. We will get to why we're doing this. Namely, any Walrasian equilibrium is Pareto-optimal. But it's also true that any Pareto-optimal allocation can be supported as a price equilibrium with transfers, as in this definition. However, to achieve the fact that any optimum can be supported, it's kind of intuitive. We may have to redistribute wealth.

And again, I have a series of lectures on this. But if we're back in the Edgeworth Box, the competitive equilibrium has picked out one particular Pareto-optimal allocation. But as there are many optimal allocations-- and to get to those, we're going to have to move this budget line up or down, hence, redistributing the wealth.

OK, so now that we've got the concept of the Walrasian equilibrium in hand, I want to focus on the primary application today, which is production in a small open economy. We have to shift gears just a little bit. It's definitely going to be general equilibrium. We're going to have J firms. And firms are producing goods.

So they're J goods because we have J firms. Firm J produces good, qJ. And there are L factors of production, labeled, in this case, Z. So Z1J is the input of the first factor by firm, J, and so on. And each firm, J, has a strictly concave production function, indicated here by f sub J. Output of consumer good, qJ, is a function of the input vector, ZJ.

So now we specify endowments. The endowments are going to be given of the factor inputs. They're owned by households. And for the sake of argument, let's say they're supplied in-elastically.

And they're not going to be consumed. What we're going to try to do is find the factor prices that's consistent with an equilibrium within this small open economy.

However, we're going to take the prices of the goods as fixed. So in that sense, it's still partial equilibrium. Think of it as a village, as a small open economy or a country, that is small relative to the rest of the world so that these price vectors are fixed.

They're not going to be the focus of our attention. We are going to, however, try to find the allocation of the inputs across all the firms, how those endowments of factors get used, as well as the Walrasian equilibrium price vector.

So firm j producing good qj will maximize the difference between revenues and costs. So if w star, say, is a candidate equilibrium price vector, the firm would choose its input vectors, Zj, to maximize revenue minus costs at that guessed equilibrium price vector, w star. p is, again, a given-- pj, the price of quantity, j, is externally specified.

So Z star j is the solution to firm j's maximization problem. And if we're finding an equilibrium, each firm will be maximizing profits and the sum of the input demands, in this case, the demand for the L-th input from firm, j, summing over all the firms j, and in particular the stars, which are already profit-maximizing, add up to the economy's endowment of the factor, Z bar, for the L-th factor.

And this is true for all, each and every, factor, L. This one is true for each and every firm, j. So this is what we're looking for, the Walrasian equilibrium in the factor markets, taking external prices as given. All right, you already know something about the solutions to the profit maximization problem. Namely, the value of the marginal product should be equal to-- of the L factor, should be equal to its cost.

Just differentiate this thing on the previous slide. With respect to zj, you'll get a derivative in the F function, and will get the factor input price on the cost side. That's what this says. Factor input price-- here's the derivative of F, with respect to the L factor. And that p is, of course, part of the exogenously-given price vector.

How many equations are there like this? Well, we have to specify here. It's the L factor. And we have L-- capital L factors all together. And this is for the j firm. But we have capital J firms. So we have L times J equations here. And we have another set of equations which are the equilibrium, that the demand for the factors adds up to the total.

But again, this is true for each and every L-th factor. So there are L of these guys. So we have L times J equations here. And we have L equations here. So the number of equations is equal to the number of variables. So in principle, we should be able to solve this system of equations to determine the equilibrium factor prices.

It's tedious and boring to keep counting the number of variables. But it is actually useful to see if we have fully specified, and we potentially have a solution. So now, we've already solved in lecture 4, which was about production, a cost minimization problem. So we defined there the cost for firm J of producing the quantity, qj, when the input vector was w.

I must say, even I went back this morning in reviewing this lecture to take a look at the earlier lecture. Today there's kind of an uneasy tension because I'm partially telling you things that you've had before, on the other hand, aware that you may not remember exactly what we had before. If you go back and look, it'll all come back to you. And some of these slides are actually repeats of what was in lecture 4.

Anyway, when we derived the least cost way of producing an output, qj, we got that marginal cost was equal to price. It's equal. Marginal cost is from your very first economics micro course, probably. Now, this second equation is the equation that the demands for the factors adds up to the total. But it's written in an interesting and a bit unconventional way.

The aggregate amount of factor l that's available is z bar l. And we want the demands of these firms for producing. But instead, we use the Shephard's lemma, the derivative of the cost of firm j, with respect to the price of the l-th factor, we derived as the amount of the l-th factor to be used. And again, you may or may not remember that that's the Shephard's lemma.

Here it is, written down here at the bottom. So we just substitute what was written as zlj, summation over j equal to z bar, we substitute in the result from Shephard's lemma to get this equation

OK, special case, 2 by 2. And there'll be another two coming later. But for now, two types of firms, J equals 1, 2-- two inputs. Now, it's useful to think of the inputs as labor and capital. And that, in fact, will anticipate the trade applications.

Are we reproducing goods with machines? Are we producing goods with labor? What happens to the labor when we have a decline in manufacturing? It comes from having more inputs.

So these are the kind of ingredients, labor and capital being kind of obvious labels for us, in the case of this two factor model. Production functions, just to write out the notation, production function for good 1, as it inputs, the second subscript denotes the firm, the first one, the input.

So this is the first input in firm 1, the second input in firm 1. f2 is output of firm 2 as a function of the first input in for firm 2, and the second input for firm 2.

And we're going to assume, in this special 2 by 2 model, that these production functions, each of them, are constant returns to scale. So it's homogeneity of degree 1. So if we're given a vector of input prices for capital and labor, we can define the cost of producing one unit of good, j.

This is called the unit cost-- or you may want to review that the cost of producing arbitrary levels just scales up or down depending on whether that other target level is higher or lower than 1. So once we have the cost of producing one unit, we can get the cost of producing any number of units.

And what inputs solve that minimum cost problem? They're denoted with a's. So for firm j, a1j and a2j are the way to achieve the least cost when the input vector is w. Input price vector is w So here I belabor a little bit what you've seen before. It's constant returns to scale. So the cost function is linear. Constant returns to scale is this linear production possibilities frontier.

Cost function is linear. Marginal cost equals average cost. If price of a good, say good j, is less than marginal and average cost, nothing gets produced. And when price is equal to marginal and average cost, then they're happy to produce anything. Although profits are 0. And price cannot be higher because they would go bananas.

And then the assertion, which I've already said twice, profits must be zero. And that's just, again, a review slide from before. So price equals cost. This is the unit cost. Price is the price of a unit. So we have cost equal to prices. OK, so another review. We did this before, too.

What's special about constant returns to scale is we can solve the problem for the one unit of output, minimize the cost of achieving one unit of output. And that's going to give us-- I guess I just said this, the cost for any other level of output, just by scaling up and down the solution. And the ratios of input use will not be changing.

I think in the previous lecture, I actually said I wasn't terribly pleased with how cryptically-worded it was. And I gave you as an informal homework assignment to do a proof by contradiction to prove that the input use ratio is going to be entirely dictated by the input prices, regardless of the scale. That is to say, this input ratio will remain constant.

And again, please, I apologize because I'm throwing at you things from a previous lecture. It was lecture 4. So that was some time ago. But I don't want to belabor it today or we won't get to something new. So for now, we'll just accept this as a fact.

Then we come to a key definition about factor intensity. Say good 1 is more intensive in factor 1 than good 2. Then the production of good 2, if naturally, firm producing good 1 uses relatively more of input 1-- input 1 relative to input 2, then does firm 2.

So the definition, to say good 1 is relatively more intensive than factor 1, means that the ratio of factor 1 to factor 2 in firm 1 is higher than it is in firm 2. So it's a pretty natural definition. Of course, it can move around. But the inequality remains the same. It's intense. No matter what the input price ratio is, the optimizing choice of inputs will always satisfy this inequality when comparing firm 1 to firm 2.

So now something a bit new that was not in lecture 4, although it's not hard. We're going to plot the cost of producing one unit of a good. We're going to plot that cost curve as a function of the input prices, w1 and w2.

So in particular, if, say, for a given w1 and w2 bar, your assertion is we're on this cost curve, then if we increase the price of the first input to w prime 1 and we're supposed to be on an isocost, meaning same cost curve, the increase in the first factor price would mean the cost would be higher.

So we have to lower the price of the second factor in order to compensate. So that on net, after we make this move, total costs are the same because it is the isocost curve.

So this down-sloping nature of the isocost curve, plotted against various input prices, makes a lot of sense. The other thing is this concave shape. This comes from a property of the minimized cost. In particular, this set of input prices at or above the cost line is a convex set. But we did this a lot with respect to the consumer optimization problem.

We defined quasi-concave utility functions and the associated concave indifference curves. And we had the weak upper contour set as being strictly convex. So these are analogs here. Now, the assertion is that this cost function is, in fact, concave. And again, as a review-- but who remembers?

We already had derived in lecture 4 that the cost function is concave in the input prices. So I just pasted in the slide to remind us. So now we have down-sloping concave.

And the other thing that's going on here is, at these particular input prices, if we go back to Shephard's lemma, the derivative of the minimized cost, with respect to, say, the price of the first factor, is the level input of the first factor, and similarly, for the level input of the second factor.

So the ratio, the slope of this line, which is the ratio of the derivatives of the cost curve at an optimum, must, by Shephard's lemma, be equal to the ratio of the optimizing inputs.

And again, logically, if you increased the input price of factor 1, keeping cost constant, you would move along this isocost line. And you would increase the amount of good 2 and factor 2, and decrease the amount of factor 1.

You substitute away from the factor, which has now increased in price. So in the earlier slide, lecture 4, I didn't replicate it. We had a Walt Disney example about machinery versus humans.

Now I'll just warn you, it is very hard to remember what's divided by what. But anyway, the fact that this perpendicular becomes steeper is consistent with the input of factor 1 going down because it's the inverse ratio. All right, did that. So now, how do we solve for equilibrium factor prices? This is just a picture of a particular firm and what it would do, firm j-- what it would do as we vary the input prices?

And we want to find particular input prices, such that both firms are minimizing costs, and all the factors get utilized. So remember, because we have constant returns to scale and because we've taken the right hand side prices as arbitrary and given, coming from nowhere, say, but certainly known to the firms, as part of the solution, the unit cost has to equal to the price for both firms 1 and 2.

So we have, from this, two equations and two unknowns, namely the factor prices, w1 and w2, that we want to solve for. So visually, we can actually put both firm 1's and firm 2's isocost lines on the same diagram.

In other words, I've gone from here, which was firm j, and firm j's particular isocost line-- talking about how it varies with w1 and w2, to this diagram, in which we have not only firm 1-- excuse me, but also firm two.

And if we're going to find an equilibrium, we're going to find price for factor 1 and a price for factor 2, each. So each firm, in an equilibrium, is going to face the same factor prices. And each firm must lie on its own unit cost curve equal to the price from this equation. So you can see the unit cost for firm 1 equal to p 2, the unit cost of firm 2 equal to p2.

These are particular isocost curves for firm 1 and firm 2 that are associated with two of the equilibrium equations. And if we're going to find common factor prices faced by both of the firms, we only want one factor price for the first factor, and one factor price for the second one. So where can that happen in this diagram? Only one spot, and that's where these isocost curves cross. So this is kind of displaying the equilibrium.

Let me say one more word about why they're shaped in this way. Remember, firm 1 is intensive in factor 1. So if we did move along its curve, lowering w2 and increasing w1-- because it's intensive in factor 1, it tends to use a lot of factor 1. So increasing the price of factor 1 is kind over big burden for firm 1.

And to compensate, we have to lower the price of the second factor quite a lot, relative to the compensation we would have to offer firm 2. For the same increase in w1, the corresponding decrease in w2 is less for firm 2 than firm 1 because, again, firm 1 is intensive in factor 1.

So this factor intensity, which I said was a very natural definition, is true everywhere by definition. Hence, the curve for firm 1 has to be steeper everywhere than the curve for firm 2. And that's why they have to cross. Questions?

OK, so again, just to review this slide, we're looking for an equilibrium. The claim is we found it now because we have found the factor prices for the first and second factor, such that each firm is minimizing its unit costs associated with the cost-- the unit cost being equal to price.

So these must be the equilibrium quantities of the factors being utilized in each of the two firms. All right, so we're kind of doing an algorithm here. The algorithm was define Walrasian equilibrium. And do that for this 2 by 2 model. And without telling you that I was going to do this, we're doing it iteratively. The very first thing we do is solve for the factor prices.

And now we're going to get the quantities this way. So the quantities, naturally, are going to come from a diagram where we have isoquants. So here we have the input of factor 1, the input of factor 2. For firm j, this is an isoquant equal production, as you move along that isoquant curve.

And again, you may remember, or not, that to minimize the cost of producing one unit, we find the tangency, where the isoquant is tangent to the isocost line. Now again, it's a bit dizzying because what's an isocost line in one space is a different-looking cost line in another.

In this space, the isocost curves are concave, as I was saying. That's in the space of factor inputs. W1 and W2 prices. w1 and w2 here, the isocost line is linear, in the space of input quantities, z1 and z2.

So again, the minimized cost of producing one unit of output here at these factor prices, w, is this tangency. And it's associated with the inputs, a1 and a2, to firm, j.

And again, the reason is something we reviewed before, that when you take the derivative of the isoquant, we're getting dz2, dz1, the quantity is staying the same. So we solve for the solution. And we're getting a relationship between the derivative of the isocost and the inputs.

OK, so now we want to solve, not only for quantities, but the equilibrium amounts. Now remember, again, here we were playing around with unit costs. But we don't know the level of the quantities yet. And likewise, hence, we don't know the levels of the inputs. Although we do know something about the ratios of the inputs one once we find the equilibrium factor prices.

So having found the equilibrium factor prices, we now turn to determining the inputs. But it almost looks like a review. The input ratio for firm 1 must be those a coefficients, the solution to the cost minimization problem in the quantity space at the equilibrium factor prices, w star. This is true for firm 1. It's also true for firm 2.

So the ratios of inputs being used must be consistent with the optimized ratio of inputs being used at the equilibrium factor prices. There are two equations there. There's another equation, namely that the inputs being used by the firm, in this case, input 1, and the second, input 2-- input 1 in firm 1, input 1 in firm 2, must sum up to the aggregate endowments.

So finally, another ingredient of the economy, the level of the input endowments, is here on the right hand side. And when we found an equilibrium in input use, it has to satisfy this system of equations. Now I can show you a picture of it. You may remember that when we did production, we talked about the production box-- not the Edgeworth Box of the consumer theory, but the production box of production theory.

We had, again, like the isoquants, input 1 and input 2 on the x and y-axis, firm 1 is oriented around 01. And we have isoquants for firm 1. And this ratio, the slope of this line, is the optimizing ratio of inputs. And again, that optimizing ratio of inputs is entirely pinned down by the equilibrium factor prices, which we already found.

So they have to lie on this line somewhere. Why here? Because when we go up to firm 2, the same thing is true, that the optimizing input ratio of factor 2 to factor 1 must lie on this line coming out of the 02 origin. So input ratio for firm 1 is fixed. Input ratio for firm 2 is fixed. Fixed at what? Fixed at these levels, as in the earlier diagrams.

Hence, in an equilibrium, to be consistent with each of them, it must be where these lines cross. So this finally determines output because we have isoquants here. So firm 1 is producing on this isoquant.

Firm 2 is producing on this isoquant. So now we've got both, the equilibrium level of the inputs-- which is boom boom and boom boom, as well as the outputs associated with these isoquants.

OK, are there questions so far? Well, let me just say, when you review, there is a lot of content in this lecture. Some of it's review, as I've already been apologizing. Go back to lecture 4.

But also, it's kind of stacked, in the sense that each segment builds on the subsequent segments. And it's very hard to take it in when I go through these slides for the first time.

But I think if you review it the second round, you'll see how the ingredients are coming together. Anyway, where are we at this point? We had a country, say, an economy, with two types of firms, j, equal 1, 2. We have two factors, say labor and capital.

We had this economy facing externally-given arbitrary prices, p1 and p2, for the firms 1 and 2. And we have now solved for the market clearing factor prices in the input markets for the level of the inputs being used and the equilibrium quantities being produced. That's how far we got already.

OK, so now I want to get to the trade part. And I'm going to add the other two. So this is two firms, two factors, and two countries. And the two countries are denoted A and B.

So each country has the same technologies. Each country can produce good 1 and produce good 2 with these factors, the two factors, labor and capital.

We could have imagined different technologies in the different countries, and in a minute, different preferences. You haven't seen the consumer side of it yet. But the famous 2 by 2 by 2 model with some amazingly cool theorems assumes identical technologies and identical preferences in the two countries. What's different across the two countries are the factor endowments. Namely, one country is capital abundant. The other is labor abundant.

So you might, for example, have thought-- although this is no longer as true as it once was for reasons of the theorem we're about to see-- that countries like Mexico or China are relatively well-endowed with labor. The US, being quite industrialized, is relatively well-endowed with capital. So the US, as distinct from Mexico, is a capital-abundant country.

And Mexico is a labor-abundant country. I choose the example carefully because, again, with NAFTA, there's all kinds of controversies about what happened to the wages of US domestic workers when the US increased its trade with Mexico. We now have a theorem that's going to tell us something about that.

Oh, anyway, here, two countries, A and B, with different factor endowments-- one being capital abundant, and the other labor abundant. Now, what if there's no trade at all? We're in autarky. What's going to determine the price of the consumption goods? Well, that's where we need the consumer part.

We're going to need the representative consumer in a given country maximizing utility, given its ownership of factors, and potentially shares. Although in this constant returns to scale world, there are no profits anyway. So we have an entirely domestic market, like China completely closed to the rest of the world.

Now we jump all the way to free trade. And if there's no cost of shipping goods around, there's an abstraction. The price of the two goods now have to be the same in each country because if there's no cost to shipping goods around, if the price of a good were lower in one country than another, then they would basically be exporting a lot of that good at a low price to the other country that's willing to buy it at a high price.

So those prices can't be different when there's no cost to shipping goods around. Now exactly where the prices are, I will show you in a minute. But first, let me give you the theorem called Heckscher-Ohlin theorem.

Suppose initially, we're in autarky. And neither country is trading. And the price of the capital-intensive good in the capital-abundant country will be relatively low-- relative to the price of that capital-intensive good in the other country.

This is intuitive in terms of the economics. You have a country that has a lot of capital and relatively little labor. That's abundant capital. That should drive down the price of the capital input, relative to the wage of the labor input-- relative also to the other country, which is abundant in labor, and should have, because there's a lot of labor around, a relatively low wage.

So those are the two statements, the price of the capital-intensive good in the capital-abundant country will be lower, relative to the price of that capital-intensive good in the other country. And vise versa for labor. The price of the labor-intensive good in the labor-abundant country will be lower, relative to the price of that labor-intensive good in the other country-- or in some, when an input is abundant, it has a low price.

Now we go to trade. We don't allow the inputs to migrate. But we do allow trade in the goods. Think about that first line. The price of the capital-intensive good is low, relative to the price of the capital-intensive good in the other country. So because it has a low price, it should be a, quote, "competitive export."

So the capital-abundant country will be exporting the capital-intensive good. The labor-abundant country will be exporting the labor-intensive good. And we're going to find an equilibrium where this kind of arbitrage is no longer profitable, where the prices will be the same. But the adjustment mechanism is coming from the possibility of exports and imports.

So let me show you an example picture. Suppose we have country B here with this production possibilities set in the two goods, G1 and G2. Now, I'm not telling you exactly why it looks like this. But it has to do with the fact that country B is relatively abundant in one of the factors. You can see, relative to country A, this country can produce a lot of good 2, and relatively little of good 1.

And that's because of the different factor intensities. The flip side of that is country A over here with its production possibility frontier. And it can produce relatively more of good 1 than good 2, in the sense of enumerating all the possibilities. If you're in autarky-- now, you remember the Robinson Crusoe economy where we had Robinson Crusoe producing two goods?

No trade-- we had, in autarky therefore, on the island, Robinson Crusoe's indifference curve would be tangent to his production possibility set. So this would be the autarky allocation in country B. Likewise, country A, with a different technology, a different island, so to speak, would have another tangency of its preference indifference curve, with a production possibility set.

Now, it's kind of easier to draw with common preferences, having the same indifference curve tangent in both. But that's not a necessary part of the argument. All we care about is tangency for country B relative to its indifference curves, and tangency for country A in autarky, relative to its indifference curves. It just happens to be the same indifference curve.

Now finally, we go from autarky to free trade. What must be the solution? Each country-- B over here will separate its consumption and production problems.

It will maximize profits by finding the point on its production possibility frontier which gives it the highest level of profits, as captured by this budget line, really-- trade line, that emanates from the new change production point.

And I know the old production point was autarky for country B. It's now shifted to be giving up some of good 1, and producing more of good 2, and then exporting good 2 and getting more of good 1 back.

So it specializes more in production. And then achieves balance in consumption by imports and exports. And country A down here is doing the opposite. It specialized more in production in the other direction, more of good 1, and then trades in the world economy, giving up good 1 and getting good 2.

And although it's not drawn here, the exports of good 1 for country A are exactly equal to the imports of good 1 for country B. So the prices of the goods associated with this external price line are the same for both countries, and have the property that, finally, we have an equilibrium in the goods markets.

In sum, we've now made the prices endogenous. P1 and 2, which were given throughout 3/4 of the lecture, are now determined internationally by free trade. All right, so we did Heckscher-Ohlin. Now we have another theorem, Stopler-Samuelson theorem.

In this 2 by 2 by 2 model with these varying factor intensities, if the price of good i increases, then the equilibrium price of the factor that's used more intensively in good i also increases, while the price of the other factor decreases. So let me say it again. And let me give the motivation here.

We're going from autarky in each country one at a time, with its internal domestic prices, to an equilibrium in the world economy with free trade at different prices. So for country b here, P1 over P2 is going down, relative to what it will be in the free trade equilibrium. And the reverse is true for country A.

So when we go from autarky to trade, we're moving the price ratio in both countries-- in different directions, but we're moving it. So let's say, what is the impact now of moving a price? If we increase the price of good i relative to the other good, the equilibrium price of the factor that was used more intensively in the production of good i will increase, while the price of the other factor will decrease.

OK, so let's go back to a proof of this in the earlier diagram, setting aside the two countries, kind of not asking for a minute where the price comes from-- except that where previously we had these isocost lines crossing, the cost lines associated with P1 and P2-- we now increase the price of good 1. So C1 equal to P1 is here. C1 equal to P prime 1 lies further out.

It kind of stands to reason that it should lie further out because we've increased the price. And we want cost equal to price. So if price is higher, cost can be higher. And cost can be higher by having higher levels of w1 and w2 as the input prices. So the isocost line for the price when the price increases, in this case, good 1's price increases, shifts outward.

We haven't done anything to the price of the other good. The C2-- where is it? The C2 line is staying constant. The C2 isocost line of price B2 is staying constant. However, we need a crossing-- that an equilibrium is described by where the lines cross. So we would move from this initial crossing to this second crossing.

And you can see, as a consequence of the increase in price in P1, that the price of factor 1-- good 1 is intensive in factor 1. That's the definition that we were given at the beginning. The price of good 1 increases.

Therefore, the input, which is used intensively in good 1, namely factor 1, should increase. And the other one decreases. So this diagram illustrates the Stopler-Samuelson theorem.

But again, in the context of free trade, those price movements come about for a reason. Namely, we've gone from autarky to free trade. So you could see earlier which way the price was moving, depending on the initial country and the autarky.

Now, although we know, again, to repeat the steps of the algorithm, we now know what happens to the factor prices from this diagram, what happens to the levels of the inputs, and what happens to output.

Well, we have this, again, box for production. We had the initial allocation ratio of the factors as quantities crossed. That is to say, the line for good 1 and the line for good 2 crossed. And they crossed here.

But now those factor prices have changed. As a consequence of the price change, the quantity of the first factor will go down because its price has gone up from country 1's point of view.

That produces a somewhat steeper sloped line. And the opposite is going on with country 2. So the new equilibrium is where these now-changed slope lines cross.

And you can actually see, in this case, that firm 1 is producing more because its isoquant will be higher. And it's using a higher level of both inputs. And likewise, the production of good 2 will contract.

So again, this factor price equalization theorem is, just to restate it, in the 2 by 2 and 2-country model, as long as we have this factor intensity assumption holding and the countries are not completely specialized, the equilibrium price of factors, inputs, only depends on the technologies and on the output prices-- determined as it may be, in such a way that the prices of the outputs equals the input costs.

But now, look, if both countries face the same prices because they trade in goods, and otherwise there would be arbitrage, then since the prices are the same in both countries, we can zero inside each country, and look at the factor prices, which are clearing the factor markets in each country. And since each country has the same technology, these equations across the two goods in each country hold.

If the prices are the same on the right hand side as a consequence of free trade, then the solution to the input prices has to be the same on the left hand side. So in short, remarkably, we have not allowed trade in factors-- no migration. But we've managed to equalize factor prices without migration simply by having trade in goods.

So that's the famous factor price equalization theorem. So vis a vis, the US and Mexico, allowing the NAFTA, free trade in goods, even with a wall, if I dare say, prohibiting Mexican migration, the theorem says that the wages in Mexico should rise in Mexico and fall to the US, to the point that the wages are the same.

So the factors bear consequences. It's not necessarily a good thing that wages of labor are falling in the US-- not good for those firms. We have to come back to those households. We have to come back to the welfare theorems to think about the winners and losers from trade. The point here only is that you can see why there is sometimes resistance to trade because trade does have consequences for factor prices.

Hence, for the welfare of the providers of those factors, depending on whether they're capitalists that own wealth, capital that's used in machinery and so on, versus laborers that have only their labor endowment.

OK, so next time we're going to go with this kind of theorem back to the Thai villages. And then we're going to go, in the subsequent lecture, to the US, and explore these ideas. OK, that's all I have for today. OK, thank you very much.