[SQUEAKING] [RUSTLING] [CLICKING]

ROBERT TOWNSEND: Let me get started by asking if there are any questions from last time. You know I really like questions.

AUDIENCE: So just to be clear, I do have a question. So just to be clear, for the pollution stuff for the last time, there are a bazillion prices, right? There's not just one price.

ROBERT TOWNSEND: There was one price for permits. The model is simple. So there's only one dimension of pollution.

AUDIENCE: OK.

ROBERT TOWNSEND: We kind of did it as a partial equilibrium problem. So the consumers would take a price as given and deciding how many permits to sell. The single firm in that example would take prices as given and decide how much to produce. But in the equilibrium, there would be a common price for permits.

AUDIENCE: OK. In that case, I'm a bit confused about something. If you have permits and stuff, if you have 1,000 people, if you have one person selling a permit, if that person sells a permit to have more pollution be polluted, then that pollution gets spread out over everyone.

So the price that someone would be willing to sell would be much lower than the actual price that person carried out pollution or something. Or am I thinking about it wrong?

ROBERT TOWNSEND: No, no, no. You're absolutely right. So that part is missing from the model because we only had one consumer or we talked about the consumer as a representative consumer and everybody wanting to do the same thing.

But you're right, there could be a potential conflict of interest if you're all supposed to pay-- well, there's something called the Lindahl equilibrium that was just going to take us to far off the path of the lecture that actually elicits from each household their willingness to pay for pollution and then adds up those numbers to get the sum of the willingness to pay.

And then, that number is, quote, "compared" to what the firms are willing to pay for the input costs. So that's the way the generalization would go.

AUDIENCE: OK, thank you.

ROBERT TOWNSEND: Yeah. It's a good question. Other questions. So just to remind you all, this is the last lecture 21, the last lecture I'm giving, although I will have class on Tuesday, which is another review session focusing mainly on the material that we've covered since the last time, trying as I usually do in those review sessions, to bring the pieces together. So that's the class schedule.

Then, for the reading list today, for the lecture today, there are these two starred papers. And one is this Abel, Mankiw, Summers, Zeckhauser paper and how it's like one or two slides in the lecture today.

We didn't spend a lot of time on it in the lecture, but it's listed there as a supplemental reading, along with my own paper. I'll spend a little more time on my own paper.

Anyway, I'm kind of treating you guys as you are, as fully functioning economists. So you should be able to read the Abel, et al, paper and understand it without me going over it this lecture.

And the previous one had that style to it. In fact, last time was the first time, I think, that I didn't finish the lecture material and asked you nevertheless to go ahead and understand the lecture and to read the starred reading. So I did not do this too, too much at the beginning of the course.

But now that you're truly trained, I think you're more independent. Anyway, feel free to ask me questions about those at any time, including next time, if it comes up because it's material that we really haven't covered much in the lectures but I started. So meaning, I do expect you to understand it.

And then, for the review sheets, we were doing the failures of the welfare theorems. The way these review questions are written, they're largely review. It's kind of like telling you what you need to know. It's hard to ask you when the answer is given, but I'll give it a try anyway.

It says, you should know well the formal statement of the first welfare theorem and conditions under which it holds. So again, I think I'll just take volunteers. Can someone tell me the roughly or even more carefully the statement of the first welfare theorem? OK. There are two welfare theorems.

AUDIENCE: OK. If I remember correctly, the first welfare theorem is something like assuming local nonsatiation and I don't think it's convexity, but there's second conditions that I forget. But to mean, local nonsatiation and something else. Then, the practical Walrasian, like, price equilibrium is pretty optimal. If I remember right.

ROBERT TOWNSEND: Yeah. That's right. It was local nonsatiation and a statement about rationality of consumer preferences. So very, very weak, which is probably why they're hard to remember. Yeah. But the local nonsatiation is there.

And then, the pollution we've already started talking about. Pollution permits. Why did we need those permits in the first place? What was it that we did wrong? Since the first welfare theorem is supposed to hold and even assuming local nonsatiation, what why did it fail in the end?

AUDIENCE: Is it really like a market for pollution? So like, people's preferences don't get accounted for there.

ROBERT TOWNSEND: Yep. Exactly. So it's a missing market problem. We assume their L goods and their markets in all of them when we do the proof of the first welfare theorem. In this case, we kind of left one out.

So there aren't prices there to guide the solution of the market to the situation where the marginal rates of substitution over households are equated to the marginal rate of production across firms. So that that's the nature of the problem.

And likewise, I thought I indicated one other. What about the second welfare theorem? Can anyone state it, including some of the assumptions that are needed?

AUDIENCE: Sure. I can take a stab at this one. So the second welfare theorem says that basically any optimal point is supported by some while raising equilibrium up to transfers. And some assumptions that we need for that are we need convex open consumption sets.

We need convex closed production sets with this concave transformation function. We need concave and locally nonsatiated preferences. And I think the last one is basically just like a budget constraint in some sort basically.

ROBERT TOWNSEND: Yeah. It's some interiority condition.

AUDIENCE: Yeah.

ROBERT TOWNSEND: Yeah. That's right. So yeah, so this one takes a bit more in terms of assumptions. And then, I showed in the lecture two pictures of illustrative examples of where things could go wrong. One had nonconvexity in the preferences, and the other had nonconvexity in production.

I don't know if it helps to give the math intuition. When we did the second welfare theorem, we were kind of getting those shadow prices by solving the Pareto problem and then essentially using them as candidates for the Walrasian or equilibrium or the equilibrium with transfers.

And those shadow prices are marginal conditions. They are derivatives having to do with differentiating the Lagrangian with respect to constraints.

Another way to put that is the candidate for prices are essentially margin utilities. Weighted margin utilities, at least in the Pareto problem, would be equated to a common Lagrange multiplier. And that's what I meant by marginal utility pricing, that the Lagrange multiplier becomes the price.

It's kind of amazing because it's all teeny tiny local. Once we find the candidate we want, all we need to do is look at the derivatives for very small movements around that point and then kind of separate the problem globally.

So it's not just that at those prices, households and firms are max-- I mean, that's true, but they're not only just maximizing locally, they could, in principle, choose things that are very far away from that local point that are global.

And that was the point of those two figures with the nonconvexity in preferences or, alternatively, nonconvexity in production, you could see how locally things look fine. But if you drift pretty far away, you find another point that either maximizes profits or maximizing utility, depending on the example.

So all of this stuff we're learning was kind of worked out by British economists and then formalized by Paul Samuelson. It's called the sort of marginal analysis way of doing economics. So that's good. So that that's essentially all I want to do in terms of reviewing from last class.

Well, the other thing-- yeah, question.

AUDIENCE: Yeah. When I mentioned the first welfare theorem, we just assumed that the preference are original and the nonsatiation. And we say that the production is a nonempty.

And we don't consider whether the production function is convex or not. But I notice that if the production function is nonconvex, we see there might not have any competitive equilibrium.

For example, when the production function is now convex, then the-- yeah, then the slide 3 showed that the point which consumers want to be maximizing utility. But at this point, it was not at the producers to maximize their profits. So in this situation, can we say that was at equilibrium? It looks like there was no equilibrium in this situation.

ROBERT TOWNSEND: Yeah. It's a good question. And it gives me an opportunity to clarify. There are two theorems. One, if a competitive equilibrium exists, and it's optimal, that's the first welfare theorem. That's the one that only needs local nonsatiation and rationality.

And the reason-- and it's true, when there's production, and there are no assumptions about production, but the intuition for why it works is you're given so much information already, namely this competitive equilibrium exists.

So what's ever going on, firms were maximizing profits, and that added up to demand and so on. So all that stuff is kind of implicitly given in the statement of the problem, if the conditionality, if a competitive equilibrium exists, then something else is true.

AUDIENCE: Oh, I get it.

ROBERT TOWNSEND: Yeah, yeah, yeah. And then, I slipped up too when I was going through those examples because I'd forgotten that I'd switched to the second welfare theorem. So the second welfare theorem is the starting point is Pareto optimal.

And I should have, when I went over those figures, emphasized how it was the single-best point in the whole space and could not be Pareto dominated, and then settled into the second part of the second welfare theorem, which is then you can maybe not decentralize if you lose the convexity.

AUDIENCE: Thank you.

ROBERT TOWNSEND: OK. I'm glad. I like questions. I know it's the last class. By the way, please come ready to ask questions next time. That's a much more fun way to review what we've done in the course than it is for me to lecture all over again.

All right. That said, I should lecture all over again, I guess. Here's lecture 21. And again, without exaggerating, this one, like the last one, is kind of exciting, both for the topics that are covered and because it's building on material that you now know that was spread out throughout the course.

I suppose a better title for this one would have been monetary economics or cryptocurrency. But Bubbles is a more appealing title. And as you will see, bubbles can exist, meaning something has value that has no value. And they can be optimal. And that also creates a scope for monetary policy.

So these things look really different from the welfare theorems that we just did. And essentially, the reason that it's happening is either that we have a version of incomplete markets or we have an infinite number of commodities.

But rather than reiterating those theorems, so this is the monetary economic slide. Let me tell you in words how we can get fiat money to have value, even when it's worth much more than the paper it's printed on.

So you have these kinds of agents moving in opposite directions. Some are moving west, as in the top line. Others are moving east, as in the bottom line.

And every period, they hit one of these trading posts. So the sequence experienced by the people traveling west, if they start with one, then next period they're going to have 0 goods, that's their endowment.

Then one good and so on. And the guys going in the opposite direction have the opposite sequence. Whenever they're able to trade at these vertical lines, which are trading posts, one agent has one and the other has 0.

And then, the basic idea is somehow the guy with 0 goods also has these pieces of paper called fiat money. And they give them up. They have value, and they get one sum amount of the good from the person who has the real commodity.

So there's a trade going on between goods and money. So we have to extend the commodity space to include these pieces of paper. And then, the guy that basically got the money is willing to take it because he or she is about to experience in the next period 0 real endowment. But they can spend the money to a person with one who was in turn willing to take it.

I'll show you this model at the end of class. And we'll work some things out. But that's an example of an economy with endowments and preferences that is pretty awful if there's no money. But it's much better with it. And then, yeah, at the end, we'll even talk about optimal monetary policy in that setup of why the Fed should be controlling the interest rate.

This is a similar model down here, if you can see it. You have people going one direction or the other. When they meet, though, they stay together for two periods. So this person's endowment goes a in the first period, 0 endowment of the good in the second period. The opposite is true. This one has 0 and then B.

So over these two periods, they can borrow and lend from each other. So this is just like a snapshot of a two-period intertemporal economy with borrowing and lending, which they will want to do because they have extreme endowment positions of the real good. But it doesn't end there.

After they're paired like that, one goes to the east, the other goes to the west. And if you think like a is a very, very big number and b is a very small number, this economy starts to look like the one above it, where they're going from large number to something almost 0, large number to something almost 0. And the person below going in the other direction has the opposite pattern.

So this economy displays both real credit, borrowing and lending and real goods, and fiat money. And of course, in the economy we live in, we arguably have both. We have fiat money, and we have borrowing and lending and credit markets.

And then, finally, we go to this third economy, where there are two locations. Agent 1 is always residing in the first location. Agent 2 is always residing in the-- Agent 4, I'm sorry, is residing in the second location. And these other two agents labeled 2 and 3 switch locations. 2 goes from 1 to 2, then back to 1, et cetera. And 3 has the opposite pattern.

You can kind of create this by taking this so-called turnpike and bending it around so that it meets like a merry-go-round. And that would create something very similar to this economy.

So much more fun, you can see at date 1 and 3, Agents 1 and 2 are paired with each other. So you can imagine that borrowing and lending, simple two-period loans, are possible. And depending on endowments and preferences, desirable.

So we have short-term credit, two-period debt, but also watch this. Now say 1 issues an IOU to 2, a promise to pay in the fourth period. 2 travels and passes it along to 4, 4 passes it along the 3, and 3 passes it along to 1, who was the initial issuer of the IOU.

And there are several other network chains like that in this economy. And those would be debt. They're private IOUs. They're not fiat money. But they would facilitate exchange and act like privately issued monies.

Now, you do have to redeem your force in the model to pay back the loan in the fourth period. But meanwhile, those IOUs circulate in the population. So this is a real economy, no fiat money. But there is a kind of money circulating private debt with high velocity.

That IOU passes hands every single period. It has the highest single velocity in the whole economy. Velocity is the amount traded per unit time divided by the stock.

Another implicit thing going on in this comparison is, depending how often agents meet with one another, you could evolve from a monetary economy with valued fiat money to the point that the credit markets get so good, especially with privately issued debt, that you don't need fiat money at all.

So don't tell the Federal Reserve. They're not going to be happy about it. But that tension is there. And I think part of the issue with digital assets and cryptocurrency is that it's kind of filling in the market and can lead us to a situation where the fiat money is no longer valued.

Well, I mean, you get the same thing in this model. a meets b for two periods, then three periods, then n periods. So they're together so much, the residual gap is not enough to create valued fiat money.

So these slides then go back to the issue of either monetary policy or distributed ledgers. We talked about distributed ledgers throughout the class off and on. Maybe that wasn't effective because it pops up in different spots, depending on the lecture.

But if you look at 1 passes it to 2, 2 passes it to 4, to 3, and back to 1, there's another path over here where 3 issues it to 4, 4 gives it to 2, 2 gives it to 1, and 1 passes it back to 3.

It turns out that you don't need both of them. But if the amount of short-term circulating debt is not coordinated, then you can run into troubles. Another way to say that is there are multiple equilibria, a continuum of equilibria, but you need to coordinate on the amount of the privately issued currencies that are about to circulate in the economy.

And if these are informationally decentralized locations, if they're not supposed to know what's going on in the other location, then there's no way they can coordinate. So one of the advantages of distributed ledgers would be, especially if you write smart contracts on them, that you can solve this coordination problem.

And I think that's where we're going with digital assets, except people aren't thinking about it yet. We're going to end up with financial crises, not because digital assets are bad, but because there's a coordination problem. Fortunately, we have a solution in the ledgers.

It's not crazy to link this to central banks because I mentioned once to you IOUs that circulated in Medieval England. Those were those willow sticks. Eventually, they were issuing pieces of paper, promissory notes as IOUs, that were circulating in the population.

And in London, those bills of exchange were-- well, all of England and especially in London, there were markets for those bills. But occasionally, they would crash. And so that's actually what gave rise to one of the first central banks in the world, the Central Bank of England, where it would buy up those bills that were falling in price.

That's the way monetary policy used to work when the Fed buys treasuries. But they may have created the wrong institution because the problem likely was the coordination problem, not the fact that the private markets were intrinsically flawed.

And the other aspect that we're going to cover today is that fiat money can be introduced. Even though it's worth nothing, it's going to have a positive value. And that's going to create other problems.

It's not like it's totally stable. It could be moving around, which brings us to the other kind of money, Bitcoin, which was introduced to be a money and to replace fiat money. That's what really made the central bank so annoyed with the whole Nakamoto experiment. They were threatened by Bitcoin. So a bit historical here. I kept it in for a reason.

When I first started teaching, I grabbed this slide, which has to do with the price of Bitcoin moving from $100 to $147 in 2013. People were already talking about how ridiculous that was. And it wasn't tied to anything. So the value was completely indeterminate. But remember, fiat money is just like that.

When you have the head of the Federal Reserve Bank of New York, Dudley, saying the problem with Bitcoin is that it has no fundamental value, don't throw stones if you live in glass houses because fiat money has the same problem. And it's just that in the US, fiat money is respected and not unstable.

Anyway, back to the $100 to $147, I can't see your faces very well, but you may be a bit puzzled because if you looked at what the value is now, it's $19,372. So over here, at the low end, the price was so low, it looks like it's 0 now when we change the scale. And then, of course, it jumped up in '17, I guess it was, and hit about $19,000.

And this is the most recent-- I kind of did this because I told you I was aware of the price three days ago was the highest on record. As of yesterday, it's back down about $200 below the highest point. But essentially, these are almost identical spikes.

Where do you think it's going to go? You want to buy? Well, what determines the value of this stuff? OK. So maybe only that it's used in exchange. Would that pin down the value? The answer is no. And now you're going to see the model.

So this is the overlapping generations model. I mentioned last time we have an infinity of goods and infinity of households. Nobody lives forever. But there's one generation after another.

We saw last time a version of that, that we could have a competitive equilibrium where the price of the second good was 1, everyone had 2 units of wealth, and they could all end up in autarky. And there was a way to Pareto dominate the competitive equilibrium.

So that is true here in this is more general setup. A little bit of a better-known setup because this is Samuelson's model, which he created at MIT, by the way. You have two-- let's simplify the lifetime. You're either young or old. So no middle-aged people. No people living 100 years. They just live two periods.

How many of them? We're going to put a little population growth in the model. So there's an initial population L0 at date 0. Then, L1 at date 1 is 1 plus n times L0. So there's a rate of population growth of n. L3 is 1 plus nL which is 1 plus n squared L1. So if the initial population is 1, then the total population satisfies this equation.

It'll be interesting and annoying because we're going to have to keep track of the number of people all the time. And we are going to want to divide by the numbers. So the algebra gets to be a pain. But I'll show you, at least in one case, how to do it. And then it will be more clear.

There's only one good period. But per period, there's an infinite number of periods. There's obviously an infinite number of people, even though no one lives more than two periods. There's a constant return to scale production technology that maps the level of the capital good, real physical savings, as it were, and the current labor supply into output of the single good.

And because it's constant, returns to scale, we know that if we multiply the inputs by a scalar, the output gets multiplied by a scalar. We do the following trick and let the scalar be the population itself. We divide by labor. So L over L goes to 1, k over L is now termed little k. Its capital per capita.

And output would scale by 1 over L. And that's like putting that L over here on the left-hand side. We have output per capita. So we can write basically a production function F, which maps output y, little y, from capital.

So there are two goods here. There's labor and capital. They're both used in production. So the capital stock is brought in from the previous period and then used in production. The marginal product of capital is going to be the rental rate on capital or it looks like an interest rate. And the wage is going to be the residual of what isn't paid out in capital.

I could have written down an explicit expression for the wage, which would be the marginal product of labor. But because we have constant returns to scale, we know there are no profits. And since there are no profits, the payments to the factors of production, labor and capital, adds up completely to output. And firms make no profits.

So these are the expressions for the wage and for the rental rate on capital. But note, that in turn, because it's a function of k, It's output less whatever the marginal return on capital was times the total amount of capital involves k, and k involves r. So you can write this as a function of r.

Here's a simple two-period max problem. Utility today plus beta times utility tomorrow. And how do they do that? They're endowed with 1 unit of labor today. They get the wage. And you can either eat it or save it, real physical savings, or call it capital stock.

Likewise tomorrow, they don't have any income. They're old. They're not working. But they do have that saving, and the saving is going to earn interest at rate little r. So whatever capital is brought in to next period, when they're old, it's going to have a rate of return of rt plus 1 for the second period because they were born at t in they're old at t plus 1.

So savings times 1 plus the interest rate would be their consumption. It kind of looks like Social Security, except here these are real physical savings, the only thing so far that they're able to do. No money, no Social Security, nothing. So we're going to get a savings function as a solution to this problem that's going to involve the prices, namely the wage, and the interest rate.

Each household will take the prices as given, as we always do in a competitive equilibrium, and solve for s. But that s is, in turn, through the production function, going to generate the interest rate. And the level of capital will also determine the wage.

So what is in equilibrium? If things are going to add up properly, then we take the savings function-- we just did this-- the savings function of an individual person at day t times the number of persons at day t, that's the population Lt.

This is total savings, which is going to be the total capital stock used in production tomorrow. And that's where the interest rate came from. It's paid out, supplying a factor of production tomorrow, namely that capital.

So this is market clearing. You might imagine from Vollrath's law, the only thing we need to worry about if we're going to clear savings equal to the capital stock, we're also going to clear the labor market and everything else. But let's just focus on the capital stock.

Now, take this equation here and divide through by Lt. Then we have little s on the right and kt plus 1 over Lt on the left. That's kind of weird because we'd want to normalize by the actual population in t plus 1, which is Lt plus 1. So this will be kt plus 1 over Lt multiplied by Lt plus 1 over Lt plus 1. That doesn't change anything. That's just 1.

But we take that thing in the denominator, the Lt plus 1, and stick it under the k, and that makes it a little k because that's the per capita capital stock at t plus 1.

After you did that, with Lt plus 1 over Lt, and we know that's already going up at rate 1 plus n. So that's where this first term is coming from. And I know it's painful to hear me go over it verbally because it's all just algebra. But this little trick is going to get used four more times. So I wanted to draw your attention to it.

The capital stock per capita needs to take into account the number of people. The number of capita, so to speak, is changing over time. That's kind of the math of this expression.

Anyway, we have little s on the right. And we already know the wage is a function of the current capital stock. The interest rate is a function of capital stock next period. Hence, kt and kt plus 1 are in this expression.

So we can effectively look for what happens if we start with initial capital stock in the whole economy at date 0, the genesis date, and stick that capital, what's going to be in, solve this equation, what's going to be the capital stock tomorrow? Well, it's meant to be-- the solution is meant to be along this line denoted as this curve.

Looks like a difference equation, mapping current into future capital. If you start with current capital, tomorrow's capital at t plus 1, at 1, would be kt plus 1. Now you do this trick. Let's go back and hit the 45-degree line and start over.

Because it's the 45-degree line, we're putting k1 down here on the x-axis. Start at k1, what capital we have in the third date, read it off the s-curve and so on. And you can see that it would evolve, if this line crosses the 45-degree line, to a steady state called k star.

So after that, the economy isn't moving anymore. There are no more dates on anything. It's completely settled down. So this is a long-run steady state of the economy, with only real savings.

Now, let's introduce pieces of paper that are intrinsically worthless. But imagine that they have value. And let's denote their value as the price Pt, and that is the price of money in terms of consumption. It's the opposite of what you would normally think. Normally, we think of consumption having a dollar price. This is dollars having a consumption price.

And I'll keep pointing that out. So if that consumption price of money is changing over time, we put Pt as 1 over Pt, that would be appreciation. That's going to be a good thing. By the way, if it were a money price, a dollar price of consumption, then having the price level go up would be a bad thing because it would be depreciating-- it would be inflation.

But here, it's a good thing because we flipped the numerator around to be the opposite. So this is a rate of return. This is a real positive rate of return if Pt plus 1 is over Pt.

And maybe people want to hold this piece of paper. If they're going to hold these pieces of paper, they have to be indifferent to that and the other way to store value over time, which was namely physical savings carried over. They would get the interest rate, the rental rate on it next period and get the savings back.

So if in an equilibrium, there's both real savings and money, then this equation has to hold. We can call the bubble, the title of the lecture, the value of money times its price. It's in real consumption terms. So if you just stick Pt plus 1 equals Bt plus 1 over n, then you can rewrite this equation too this way because n is constant.

I'm not manipulating the amount of money in the economy. I'm keeping it constant, although the price is moving. So then, we have from 2 this equation about how the bubble, the real value of the bubble is changing over time. And it's related to the marginal product of capital.

Furthermore, I could divide by the population. We'd have a bubble per capita. And I'll do the same trick I just did before, with apologies, but I'm not going to go through the algebra. Take the Lt, divide through, then you have a mismatch between t plus 1 and t.

Then have Lt plus 1 divided by Lt plus 1, parse it out, and you get this thing, that the real per capita bubble is moving over time, according to the marginal product of capital.

But that arithmetic, which over here created the 1 plus n hanging out there, that extra term, that 1 plus n is in this math as well. And you're just dividing by it. So that's where the n is coming from, that rate of population growth.

This is all a big premise right now. We're just saying, if there were money, if it had value, the value of it has to be increasing at the rate that they can earn money in another way, namely by storing. And so you're getting an equation for how the value of the bubble, hence money, must be evolving over time.

Now, savings can be done in the two ways, as I've kept saying, but here's the equation. When they're young, we have the savings function of the young people, depending on the wage and tomorrow's interest rate, times a number of young people. But they can save in two ways. They can save in the real bubble or they can save in the capital stock.

And now, again, doing that same math as always, start solving for Kt plus 1. Subtract B off of the right-hand side. Then divide through by the appropriate labor term, and you get this expression. That's the third time we're doing the algebra. It should kind of be obvious where it's coming from because Kt plus 1 is equal to Ls minus B. And the rest of it is finding that end term and dividing through by that term.

So now, this is just another way of saying that the amount of capital next period is what it would have been before in the real economy without money, except now we have bubbles of money that have value. So we've got to subtract off the bubble. That's kind of preventing the capital stock from being as large as it would have been.

So 3, this thing, gets rewritten at the top of the next slide to be the bubble b So now solve this thing for b, little b, bubble per capita. It's this thing, and that's one equation. And we also know something about bubbles over time. As I was saying, we did that. That was this thing at the bottom of the slide before last.

So we have those two equations. This has to be true in a steady state. Should emphasize. There are t's all over the place here, but in a steady state, we're not going to have any evolution. So the t's are going to disappear.

So Kt plus 1 will equal Kt. There'll be no t on the b, Kt, blah, blah, blah. So when I rewrote 3 hurriedly just then, I rewrote it without the t's. And that's what this thing is at the top. And with the t's, we have this thing, the second term, which I've just pasted in from previously.

So if we're going to have a steady state, then these b's can't be moving around. So bt plus 1 over bt has got to be a constant. And if the left-hand side is the same, 1, then the right-hand side has to be the same, 1.

Hence, the marginal product of capital has to be equal to the rate of population growth. So if there is a steady state with a real bubble, then the rate of population growth has to be equal to the marginal product of capital, this thing at the bottom here.

There's a couple of other ways to see this. One, take this equation, but let me rewrite it verbally for you. We have output coming from the production function. Take this k and put it on the right-hand side. You can always eat the capital.

You remember that medieval village economy with seed and storage and all of that, where we talked about the evolution? So you can always eat this, the harvest, or you can save it. So it's like that. So this k can be eaten. So you have total resources available today to be f of k plus k.

And how do you use that stuff? You can eat it or save it. Eat it today, that's c. Tomorrow, tomorrow's k, would be k, except darn it. We got that population growth again. So if we're going to keep k constant in a steady state, the resource requirements to have a constant per capita capital stock tomorrow are that you have to augment the capital per capita today by n. Same trick, fourth time.

So anyway, writing it that way, we want to maximize consumption. So you solve consumption equals to f of k plus k minus k times then 1 plus n. And where is that object, that c maximized? It's maximized by finding the first-order condition, and that's exactly where f prime of k is equal to n.

So that's so-called golden rule. Distressed to see this morning that this k star is not, however, the previous k star. This is a new golden rule k star. Should have called the other k autarky for the economy without money. In this case, star would be the golden rule k for the steady state bubble equilibrium.

So it doesn't have to happen that you will be at the steady state. If k were greater than k star, everyone could be better off. Why? It's as if, if you started at k greater than the golden rule, the economy's overaccumulated the capital. And then to maintain a constant per capita capital stock, you've got to take something out of production, not eat it, and squirrel it away for tomorrow.

You could actually do better by increasing consumption of the current generation, having less capital stock to maintain, and then going out to the infinite future. None of the future generations are going to be worse off.

This experiment is exactly like what we did last time with the infinite number of goods economy, where we can engineer a one-time improvement in at least one of the agents by creating something from nowhere. Basically, we move the overaccumulation of capital into the current generation and let them eat it. And then, subsequent generations don't suffer either because they don't have to try to maintain this per capita capital stock, which was too large.

So is it possible that the economy can overaccumulate capital? Yes. So we're going to look at two steady states, the previous one without any money and the new one with the steady state bubble. An interesting thing about the steady state bubble, this one that we've already described, is that it has the property that it achieves the golden rule.

And that's because-- you can kind of see it here-- when the bubble is constant and not depending on time, then f prime of k must be equal to n. So the golden rule is achieved in the steady state of the bubble equilibrium.

And that was this top line. In a steady state with bubbles, the golden rule is achieved. And that's going to be this point. So that's k star, that's the golden rule k, corresponds to a particular bubble that solves the other equation. That's called a b star. This is what we would like to get to.

And what happens in the economy with no money? You're down here at essentially an autarky value. So this is a perfectly fine competitive equilibrium. There is no money. But in it, the economy will have overaccumulated capital.

And a Pareto improvement can be engineered by reducing the amount of capital. How does that happen in a steady state? You give other people a way to save in worthless stuff. So instead of real physical savings, they're scrambling to get these pieces of paper.

The rate of return on the pieces of the paper has to be the same as physical savings. It has no intrinsic value. It has an extrinsic value because the price of money is positive. So in other words, these perfectly worthless pieces of paper, fiat money, have a value in a competitive equilibrium. And the money's actually helping to recover the Pareto optimum.

So you see the connection to the welfare theorems here. We have an infinite number of people, an infinite number of periods. The welfare theorems fail. If you start adding up the value of wealth in the autarky equilibrium, it goes to infinity like last time. That's why the first welfare theorem fails.

And the way to recover optimality is to create these worthless pieces of paper and let them trade. It's totally amazing. But it's an intrinsic part of money. However there's a downside here, which is it's unstable. So let me just describe what's going on without expecting you to be able to reproduce all the math. You're plotting, this was the autarky value.

This is the golden rule value that we want. Now we want to look at starting from different points with the economy evolved to one or the other of those points. And this is an equation, depending on whether you're above it or below it for the capital stock to evolve, this equation, the BB, bubble bubble equation, has to do with whether you're to the left or the right of it, whether the bubble is evolving.

And these arrows denote all the forces in the underlying economy. And if you're lucky and you start away from the golden rule steady state, you would evolve on a knife edge and eventually converge to it.

So this is the good news. The economies that start with an appropriate amount of bubble or capital will evolve to this optimal golden rule, so steady state. But this is the really bad news-- this economy can also evolve to real savings and no money. And money inflates away. It could start with a positive value. This is an equilibrium. And it doesn't require stupid people. This is a fully rational expectation dynamic path, where the value of money goes to 0.

OK, well, so why did I show you that? I can replace the word fiat money with cryptocurrency and ask what value it ought to have. It's perfectly plausible to me that you could see the price moving around. Maybe it will go to some constant value if this Bitcoin is also useful in exchange, the way money is in that overlapping generation's model.

Maybe it's going to it's going to collapse again, and the price will go to 0. All those phenomenon are perfectly consistent with Bitcoin playing a useful role in exchange but not able to find a stable value.

But again, the point is, and actually we've been emphasizing this, starting with fiat money the way Samuelson did, it's a property of any money. It's a property of fiat money and so on. The Fed worries a lot about forward guidance. They're constantly telling people what they predict to happen in the future so people don't panic.

If you think fiat money is not going to be worth what it is in the future, you're not going to want to hold it. Oh, that's one explanation that traders have for what's going on with Bitcoin, namely, there's just an ever-increasing amount of government debt in the economy. The Fed has bought up all those securities effectively and issued accounts at the Federal Reserve in worthless pieces of paper.

Are you scared now? So the market may be anticipating that, in fact, there aren't enough real resources to pay off the debt. The only way that the Fed and European governments are going to pay off the debt is to inflate. And if the market starts to expect inflation, fiat money is less valued, and they're shifting into Bitcoin.

So that's what traders think is going on. I switched the story on you a bit because now we have fiat money competing with Bitcoin and vice versa. But I think you could imagine how you could get both in the model.

So the way you get both in the model is to set up an underlying economic environment and have gaps. Say autarky households aren't smoothing, there's a lot of fluctuations, blah, blah, blah, blah, blah, fiat money can help smooth. But maybe fiat money doesn't smooth it out entirely. And then, you're left for tokens like Bitcoin to play a role on top of fiat money.

So you could actually have both in the model. It's not true that one has to dominate the other. They could both coexist. And partly what we're seeing in these cryptosystems is crypto coins are a way of transferring fiat money around.

You just create tokens that get transferred around. But they're claims on the underlying fiat money. But the tokens can have value because they facilitate exchange, even if they're not backed by the fiat money. So we could get both of them in.

Now, this is pretty abstract, this overlapping generations model. You might say it's totally unrealistic. By the way, it's another interpretation of Social Security with pay as you go, namely the young pay in to Social Security, but it's not sitting there in some fund or something.

It's not in a lockbox. It's just flowing right out to the old generation. So people have criticized Samuelson's model as a not realistic model of money because of the extreme assumptions about finite two-period lives and so on.

But could it be applied? Well, these guys, Abel, Mankiw, Summers, and Zeckhauser said, look, we could actually look to see whether economies are on an efficient or inefficient path. And I thought I had started this on the reading list. So anyway, you can read this. Yeah, yeah, I did. It's on the reading list. You should be able to understand it. This, you won't understand. It's just explaining what they did.

They didn't find any inefficiency, but a subsequent person, Geerolf, did. He adjusted what those guys did to take out land rent, having profits of entrepreneurs divvied up into opportunity cost wages. And he actually finds that Korea, Taiwan, Singapore, those East Asian growth economies, with all that capital, they were actually inefficient. They were accumulating too much capital. They were beyond the golden rule. It's really fascinating. All right.

The other part is what I was alluding to at the beginning. If you create more and more real financial claims that can be traded, then fiat money can also be vulnerable. And you could convert those economies with money to economies without them.

So when you have Bitcoin and tokens and cryptocurrencies and especially digital assets, which are claims on real capital circulating in the economy as money, you may end up with that model where there is no valued fiat money. And again, that would threaten central banks. But it doesn't mean it's a bad thing. In our terminology, it could be going toward a new Pareto-optimal allocation.

Other ways to get money in the model, you could require it to pay taxes. So even if there's a last period, everyone has to pay it in, otherwise it could have 0 value. So the very first monetary models were finite-period models, but in the last period, people have to pay taxes in money.

Some governor in Ohio got a smart idea which, is I guess he liked technology. He is allowing Ohio citizens to pay their state income taxes in Bitcoin. Maybe that'll stabilize the price. And you could require that money be used for goods. We have not been writing models like that.

This is like fiat. Literally, by law, you're required to use money. And you have to hold it and have to have acquired it earlier in some way. These are called cash in advance models. So if you have certain types of goods in the economy that require cash, and those are valued goods with a margin utility of infinite at 0, then the value of cash isn't going to fall to 0 because if it were, you'd never be able to buy those goods. So that's kind of a way of, again, a mechanical way of making sure the value of money doesn't go to 0. Like it or not, I'm exposing you to different modeling strategies.

So let me come back to the starting point, which are these agents that go 0, 1, 0 and 1, 0, 1. This is a spatial model of money. Why is the money going to be valued here? Because they're not all together to trade with one another at some initial date over all these future paths of consumption.

They can only trade with each other when they meet. So this is an extreme version of incomplete markets. And you know what can happen to competitive equilibria with incomplete markets. They don't have to be optimal.

Specifically, let's write down the problem of an agent of type of I. If you can see this. It's a little bit tiny. We have discounted utility of consumption. Consumption and money are non-negative. And we have this budget equation, which reads like this-- they have some endowment, maybe positive. This is now the dollar price, by the way, of goods, the usual price.

They have the valuation of their endowment plus the money they brought in from the previous period. They may have to pay some lump-sum taxes. But then the residual can be used either to buy consumption or to take money over to the next state. So again, this is like sources and uses.

And what's the solution to this? Solution, by the way, by manipulating the money. The way they determine how much they're eating today or eating tomorrow is to either cash out the money or acquire it and store it and carry it over.

So not too surprisingly, you might expect, and it's true, that the marginal-- let me write it down here-- that with one exception, the marginal rate of substitution in consumption, of consumption at t minus 1 for consumption at t, that ought to equal the price ratio.

So remember your basic lectures, consumer theory, two periods, money is a way to carry value over from one period to the next. It's like borrowing and lending, except that it appreciates or depreciates depending on the price level.

So you would expect this equality to hold, that the tangency involves the marginal rate of substitution and consumption on the indifference curve equal to the price ratio implicit in the budget model.

Now, I was covering up the inequality because it could be that you're holding money. The marginal utility of consumption today is greater than the marginal utility tomorrow discounted. So you'd like to eat more today. So you start driving that money down, and the money hits 0, and you can't go negative. And still, the margin utility today is greater than the margin utility tomorrow.

So these guys actually-- where did it go? These guys here experienced that. They start at 1. They acquire money. They're going to have 0 next period. They start dumping the money, but they run out, and they can't go negative. So for them, they're at a corner.

Guys that come in with money and spend it, because otherwise they have no endowment at all, are nevertheless not going to have enough money. So we can look for an equilibrium where for some periods and some types of agents, this is at a strict inequality, namely the guys that are hitting 0. And the other ones 0 endowment. And the other ones, it is that equality.

Let me draw you the picture. So even though it's an infinite horizon economy, we can reduce it to a two-period economy and guess what the steady state equilibrium is going to look like. Guys that start with 1 unit would be here. If they just ate their endowment, they would have consumption of 1 today and none tomorrow.

And then, moving along the budget line, at constant prices, they get money today, giving up consumption good today, and get money tomorrow. If the price level of money tomorrow, price of goods tomorrow is the price of goods today, then they're moving right along this budget line. And where would they end up? Well, here's an indifference curve. Here's a tangency. It's on the budget line.

Now, how did I know it would be there? Namely, look at this expression. This is the marginal utility at t minus 1 to t or the marginal utility at t to t plus 1. There's a beta down here. Tomorrow doesn't count so much. Beta is between 0 and 1. So the utility, margin utility tomorrow, is always less than the margin utility today. The only thing that could compensate for it is the price movement.

But if we start with constant prices, then this expression is only going to hold if we're able to adjust consumption. We want to get the margin utility up. Consumption at t is greater than at t plus 1 in order to get the margin utility up to the point that the marginal rate of substitution is equal to the constant price ratio.

Now, is this a good thing or a bad thing? Well, it is going to be a monetary equilibrium. The guess is good. So the path would be guys with 1 are going to go to c star and then c double star, then c star, then c double star, with money moving in the opposite direction from something positive, when they have 1 they acquire it, to 0, hitting the corner, back to positive amount again.

So money is going to alternate over time at the individual level. Consumption is going to alternate over time. The money is going to have value at a constant price. But unlike the overlapping generations model, this one is not a Pareto-optimal allocation, even though we have money.

You actually know the answer already. Let me remind you. How do we determine a Pareto optimum? Well, let's all the guys that start with, say, 0 be called A types, and all the guys that start with 1 unit of the consumption are going to be called B types.

And then, maximize the weighted sums of utility. We usually have lambdas here. I apologize. Now, they're w's. This is the weight on type A. wB is the weight on type B. We maximize the weighted sums of expected-- there's nothing random-- discounted utility for both of them.

We get a first-order condition. You remember this stuff? We've done this before. This is the Pareto problem. Weighted margin utilities are going to be equated over agents and equal to a constant Lagrange multiplier. You've seen this before.

This is a new application, that's all. So the marginal rate, solving this equation, the marginal rates of substitution over the agents have to be the same. But does it pin down the allocation? No. We're in an Edgeworth box. There's one good in the aggregate available today, one good in the aggregate available tomorrow.

We know the Pareto optimum has to give all the guys equal consumption. You can satisfy this by having type A having constant consumption, type B having constant consumption, but those consumptions don't have to be equal to each other because things are going to cancel out of the numerator or the denominator.

So this picture, marginal rate of substitution for day t and day tau is equal to each other for agents type A and type B. But to review, right, you remember this lecture when we did the Pareto optimum and your Edgeworth box? You can be anywhere on the contract curve, and it's all Pareto optimal. Anything that's off the diagonal is not Pareto optimal in this environment. And this one is off the diagonal. So it's not Pareto optimal.

Where do you want to go? You kind of want to go here. And how would you engineer that? You have to generate a deflation so that they're getting a positive real rate of return on money. That's going to make them want to hold more money and eat less today. And that's going to put them at a tangency up here where they have equal consumption.

So there is a Pareto-optimal allocation. It's a monetary equilibrium, but it's an equilibrium with an activist monetary policy that's withdrawing enough currency out of the system in lump-sum taxes to create a deflation.

Deflation isn't bad here. It means that the purchasing power of money is going up. So that's all the rest of these slides I don't have time for. You are able to achieve these Pareto optimal allocations but only with an activist monetary policy, which is deliberately manipulating the amount of currency in the system.

Questions. So welcome to the land of monetary economics. It's a lot of fun. So now you're able to see in this last lecture how real financial markets are linked to money markets, the role of cryptocurrency, controversies involving its price, and how we as economists can think about these values, and actually how we would recommend monetary policy.

And I will add that if there was cryptocurrency, you would want an activist amount of manipulation of the amount of cryptocurrency in the system for the same reason that I just described to you.