Recitation Practice Problems: Solutions

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1 Theory of Insurance Markets

The houses in the mountain-top town of Conway are at constant risk of lightning strike. Each of the 100 residents of Conway has a 1 in 100 chance of having his or her house ever struck by lightening. Each lightening strike does $50,000 in damage. Lightening strikes are independent events, and lightening only damages the houses it directly hits. Each resident of Conway is a risk averse expected utility maximizer, with utility of wealth U (W) = ln W . In addition to his or her $50,000 house, each resident has $100,000 in wealth.

1. Without insurance, what is the expected wealth of residents in Conway? What is the certainty equivalent of wealth for the residents of Conway?

- *The expected wealth of the residents in Conway is given by:*

$$E[w] = 0.99(150,000) + 0.01(100,000) = 149,500$$

- *The expected utility of the residents in Conway is given by:*

$$E[u(w)] = 0.99ln(150,000) + 0.01ln(100,000) = 11.9143$$

- *The certainty equivalent is the amount of guaranteed wealth that makes the consumer indifference between the current risky situation and the certain income. This is given by*

$$11.91 = ln(CE) \rightarrow CE = 149,393$$

- *Note that since the residents of Conway are risk averse, the CE is below the expected wealth.*

1. Suppose that 2 of the residents of Conway are in one family, and they decide to form a mutual insurance company. They agree to fully share the cost of rebuilding each house struck by lightning. Are the members of this family better off as a result of this plan? What insurance principle is operative?

- *If* n *residents join the mutual insurance company, they will each pay* $\frac{50,000k}{n}$*, where* k *is the number of houses that are struck.*
- *If 1 of their houses are struck, they will pay* $\frac{50,000}{2} = 25,000$*. If 2 houses are struck, they will each pay* $50,000$
- *Each person in the co-op has a 1% change of being struck, so there is a* $100 * \frac{1}{100} * \frac{1}{100} = 0.01$ *percent change that they both get struck, a* $100 * \frac{1}{100} * \frac{99}{100} * 2 = 1.98$ *percent chance that one of the two get struck, and a* $100 * (\frac{99}{100})^2 =$*98.01% chance that none of them get struck*

- *Each family member, therefore, has an expected wealth of:*

$$E[w] = 0.9801(150,000) + .0198(125000) + 0.0001(100,000) = 149500$$

- *Each family member has an expected utility of*

$$E[U] = 0.9801 ln(150,000) + .0198 ln(125000) + 0.0001 ln(100,000) = 11.9147$$

- *The certainty equivalent of this gamble is* 149,453*, which is slightly more than in autarky. So they weakly prefer this, even with only 2 people!*

1. Should the family open up the mutual insurance plan to all other residents in the village?

- *Suppose now that everyone in the village could buy the insurance - in this case, each member would pay* $\frac{50,000k}{100}$*, where k is the number of houses that are struck. If 1 house is struck, they will pay $500, if 2 are struck, they will pay* $1,000*, etc.*

- *The probability that no houses are struck is* $(\frac{99}{100})^{100} = 0.366$*. The chance that only 1 house is struck is* $(\frac{99}{100})^{99}(\frac{1}{100}) * 100 = 0.369$*, in which case you pay $500. The probability that 2 houses are struck is* $(\frac{99}{100})^{98}(\frac{1}{100})^2 * 4950 = 0.1848$*, in which case you pay $1000. The probability that 3 houses are struck is* $(\frac{99}{100})^{98}(\frac{1}{100})^2 * 161700 = 0.0609$*, and so on....*

- *You can see here that there is a significant chance that you have to pay something higher than $500, but there is a very tiny chance you have to pay the full loss yourself.*

- *The expected wealth of the arrangement is*

  $$E[w] = 0.366*150000+0.369*149500+0.1848*149000+0.0609*148500+...+10^{-20}*100000 = 149500$$

  (take my word for it!)

- *However, since the utility function is concave, this is going to yield a higher utility than the 2-person insurance scheme. This is the power of risk-pooling - you increase the probability you have to pay something to decrease the probability you have to pay a lot.*

1. State Farm insurance decides to offer actuarily fair insurance to the residents of Conway. What price do they charge for the insurance? Would the residents of Conway prefer to buy the State Farm insurance (i.e. do they prefer it to their mutual insurance company)?
   - *Actuarily fair insurance is priced at the probability that each individual house is struck, times the loss if it is struck, which in this case is $500.*
   - *This is very similar to the case above, except that there is now a 0 probability that you have to pay anything higher than $500 and a 100 percent change that you have to pay $500. These is NO risk in this case.*
   - *The expected wealth is*

     $$E[w] = 149500$$

   - *Since there is no risk, and they are risk averse, they prefer this!*
   - *This is an example of risk transfer (from the town to the insurance company)*

2. Consider, instead, the case where there is a 1% change that a really bad storm will come and strike all of the houses in Conway at once. Will the mutual insurance plan in part (3) be welfare improving in this case?

- *No, they can no longer pool risk because they are all facing exactly the same aggregate risk.*

2 Regression Discontinuity

A key policy question is whether the benefits of additional medical expenditures exceed their costs. The tendency for patients in worse health to receive more medical inputs complicates empirical estimation of the returns to medical expenditures Almond, Dyle, Kowalski, and Williams (2010) study this by looking at the health outcomes for newborn babies. Hospitals classify babies as "very low birth weight" when they are just under 1500 g, and use the classification to trigger additional treatment (NICU stays, diagnostic ultrasounds, etc.). They are able to see a baby's birth weight as well as whether they lived through their first year. They would like to know what the *causal* effect of this additional treatment is on health outcomes.

1. How would you set up a regression discontinuity design in this setting?

   - *You would look at the health outcomes (i.e. survival probabilities) of babies with birth weights around the cutoff. Right around the cutoff of 1,500 grams, babis should differ only in their probabilities of receiving additional health treatments and not in their underlying health.*

2. Doctors tell you that a baby's survival probability is increasing in its birth weight (i.e. babies born at 4 pounds are more likely to survive than babies born at 3.7 pounds, regardless of the medical treatment they receive). Is this a problem for your design?

   - *In this case, it is going to be important to control for the birth weight of the baby, but it is not a problem for the research design. In fact, we can even allow for there to be a different slope (i.e. relationship between birth weight and health outcomes) on either side of the line.*
   - *If we did not take this trend into account, we would be underestimating the effect of the medical treatment. Babies on the left of the threshold are more likely to die in their first year in the absence of the treatment (call this effect x). If we didn't separately control for that effect, we would estimate $y = z + x$, where z is the true treatment. Since x is negative, y would be less than z.*

3. Nurses know about the additional procedures conducted when the baby just under 1,500 grams. Would it be a problem if some nurses mis-recorded some babies' birth weights to get them just into the "very low birth weight" category?

   - *Yes, this could be a problem for us. We are making the assumption that babies on the left and right side of the threshold are the same other than the medical treatment that they receive in the first year. If, for example, nurses are more likely to do this when the parents are very informed and agressive, and aggressive and informed parents also are better and following up on other care within the first year (i.e. taking medications, bringing them in for follow up visits, etc.), then the babies on the left of the threshold would differ from the babies on the right of the threshold in more ways than just their treatment. This would invalidate the design.*
   - *Note that if the mis-recording was entirely random, that would be okay.*

4. Suppose you find that being just classified as "very low birth weight" causally increases the baby's medical costs increase \$4000 and increases the probability that she lives by 1%. You conclude, given reasonable estimates of the VSL for this population, that the benefit to this additional spending outweighs the cost. Should you recommend that hospitals give all babies this additional medical treatment?

   - *Note that the RD estimate is a local estimate, meaning that it s the causal estimate of medical treatment for babies at a birth weight of 1,500 grams. It is likely in this case the causal effect of this medical treatment is lower for babies with a higher birth weight (for example, precautionary ultrasounds are less likely to catch things in healthier birth weight* *babies). Therefore, you sould not generalize your estimated causal effect to the general population.*