Recitation 3: Consumer Theory and Food Stamps

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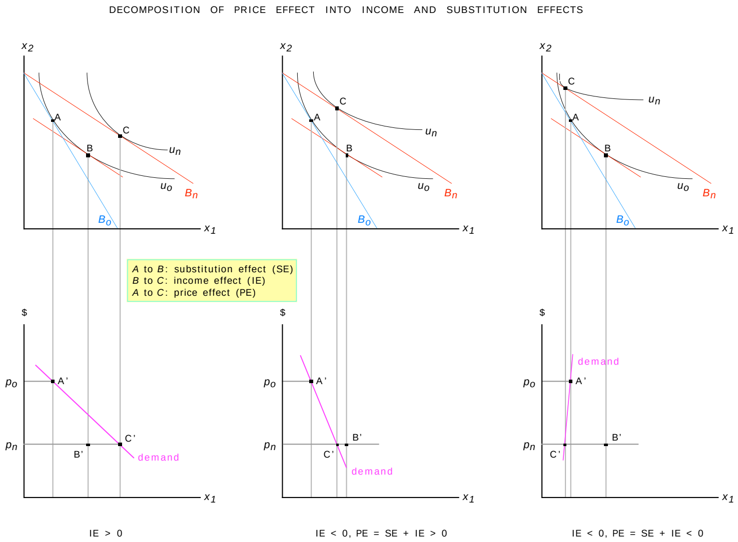
Outline For Recitation

1. Review of Income and and Substitution effects and demand curves
2. Example Problem: In-Kind Transfers
   - Simple Problem: Choosing food expenditure subject to budget constraint.
   - Policy 1: A tax credit for each unit of food
   - Policy 2: Food Stamps

Review of Income and Substitution Effects and Demand Curves



Example Problem - Setup

- 2 “goods”: Food (F) and all other spending (G)
- Income: $90
- Price of Food = $1 per unit
- Price of all of goods: $2 per unit
- Consumer preferences are Cobb-Douglas and given by:

$$U(F,G) = F^{\frac{1}{3}} G^{\frac{2}{3}}$$

- How much F and G will the agent consume?

Primal Problem: Choosing F and G to maximize utility, subject to Budget Constraint

- Budget Constraint: $P_G G + P_F F = I$ or $F + 2G = 90$.
- Lagrangian is:

$$L = F^{\frac{1}{3}} G^{\frac{2}{3}} + \lambda(I - P_F F - P_G G)$$

- *Handy Trick - apply monotone transformation (i.e. take logs).*
- 3 first order conditions (with respect to F, G and λ)

$$\frac{\partial L}{\partial F} = \frac{1}{3F} - P_F \lambda = 0$$

$$\frac{\partial L}{\partial G} = \frac{2}{3G} - P_G \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = I - P_F F - P_G G = 0$$

$$\rightarrow F^* = \frac{I}{3P_F} = 30 \quad G^* = \frac{2I}{3P_G} = 30$$

- These are the *Marshallian* demands

Indirect Utility and the "Dual" Problem

- Indirect Utility: Utility consumer can attain given a budget and prices

$$U(F^*, G^*) = \left(\frac{90}{3}\right)^{\frac{1}{3}} \left(\frac{90}{3}\right)^{\frac{2}{3}} = 30$$

- The dual is to minimize expenditure in order to attain a given utility level

$$L = P_F F + P_G G + \lambda \left(30 - F^{\frac{1}{3}} G^{\frac{2}{3}}\right)$$

- 3 first order conditions (with respect to F, G and λ)

$$\frac{\partial L}{\partial F} = P_F - \frac{\lambda}{3F} = 0 \rightarrow \lambda = 3FP_F$$

$$\frac{\partial L}{\partial G} = P_G - \frac{2\lambda}{3G} = 0 \rightarrow \lambda = \frac{3GP_G}{2} \rightarrow F = \frac{GP_G}{2P_F}$$

$$\frac{\partial L}{\partial \lambda} = 30 - F^{\frac{1}{3}} G^{\frac{2}{3}} = 0$$

$$\rightarrow F_h^* = 30 \left(\frac{P_G}{2P_F}\right)^{\frac{2}{3}} = 30 \quad G_h^* = 30 \left(\frac{2P_F}{P_G}\right)^{\frac{1}{3}} = 30$$

Policy 1: Tax subsidy for Food Spending

- A subsidy of 0.50 for each dollar spent on food for these households.
- Budget constraint becomes:

$$1 - \frac{1}{2} \quad F + 2G = 90$$

- Two questions:
  1. What are they going to consume now?
     - What change comes from the **substitution** effect?
     - What change comes from the **income** effect?
  2. What lump sum of income could we have given them such that they are indifferent between the lump sum and this subsidy?

Primal Problem: What are they going to consume now?

- Lagrangian is:

$$L = F^{\frac{1}{3}} G^{\frac{2}{3}} + \lambda \quad 90 - \frac{1}{2}F - 2G$$

- FOCs are:

$$\frac{\partial L}{\partial F} = \frac{1}{3F} - \frac{1}{2}\lambda = 0$$

$$\frac{\partial L}{\partial G} = \frac{2}{3G} - 2\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 90 - \frac{1}{2}F - 2G = 0$$

$$\rightarrow F_{\tau}^{*} = 60 \qquad G_{\tau}^{*} = 30$$

Primal Problem: What part of this change comes from the substitution effect?

- Note that we just moved along the Marshallian (uncompensated) demand curve above.
- The substitution effect comes from moving along the Hicksian (compensated) demand curve.

$$F^*_{\tau,\text{SE only}} = 30 \left(\frac{P_G}{2P_F} \right)^{\frac{2}{3}} = 47.6$$

$$G^*_{\tau,\text{SE only}} = 30 \left(\frac{2P_F}{P_G} \right)^{\frac{1}{3}} = 23.8$$

- The different between the two answers is the income effect! It increase your consumption of both.

Dual Problem: What lump-sum transfer could we give them so that they are indifferent?

- What utility level do they achieve with the tax subsidy?

$$U(F_\tau^*, G_\tau^*) = 60^{\frac{1}{3}} \times 30^{\frac{2}{3}} \approx 37.8$$

- So, now we want to find the income they would need to get this utility at the pre-subsidy prices. We can do this using the indirect utility function:

$$U(1, 2, I) = \quad \frac{I_{\text{lump sum}}}{3}^{\frac{1}{3}} \quad \frac{I_{\text{lump sum}}}{3}^{\frac{2}{3}} = 37.8$$

$$\rightarrow I_{\text{lump sum}} = 113.4$$

- Therefore, they need $113.4 - 90 = \$23.4$ to make them indifferent.
- With the subsidy, the government paid $0.50 * 60 = 30 > 23.4$!

Why is the lump-sum transfer cheaper?

- Why do you have to give them less? Because they have diminishing MRS!

$$F^*_{\text{lump sum}} = \frac{113.4}{3} = 37.8$$

$$G^*_{\text{lump sum}} = \frac{113.4}{3} = 37.8$$

Policy 2: Food Stamps

- What does the budget constraint look like with food stamps?
- Who is indifferent between food stamps and a lump-sum transfer?
- Who prefers a lump sum transfer to food stamps?
- Who prefers food stamps to a lump sum transfer?
- What would the budget set look like if you could sell food stamps on the black market for half their value?

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