

# Lecture Note 17: The Market for Risk

David Autor, MIT and NBER

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# 1 Introduction

In the previous lecture note, we developed a model for risk preference. Now we're going to put it to work. We will first consider what individuals will be willing to pay to defray (avoid or reduce) risk. We will next consider how markets for risk (AKA insurance markets) work in practice.

## 2 Risk aversion and insurance

- Consider insurance that is *actuarially fair*, meaning that the premium is equal to expected claims:  $\text{Premium} = p \cdot A$  where  $p$  is the expected probability of a claim, and  $A$  is the amount that the insurance company will pay in the event of an accident.
- How much insurance will a risk averse person buy?
- Consider a person with an initial endowment consisting of three things: A level of wealth  $w_0$ ; a probability of an accident of  $p$ ; and the amount of the loss,  $L$  (in dollars) should a loss occur:

$$\begin{aligned}\text{Pr}(1-p) & : U(\cdot) = U(w_0), \\ \text{Pr}(p) & : U(\cdot) = U(w_0 - L)\end{aligned}$$

- If insured, the endowment is (incorporating the premium  $pA$ , the claim paid  $A$  if a claim is made, and the loss  $L$ ):

$$\begin{aligned}\text{Pr}(1-p) & : U(\cdot) = U(w_0 - pA), \\ \text{Pr}(p) & : U(\cdot) = U(w_0 - pA + A - L)\end{aligned}$$

- Expected utility if uninsured is:

$$E(U|I=0) = (1-p)U(w_0) + pU(w_0 - L).$$

- Expected utility if insured is:

$$E(U|I=1) = (1-p)U(w_0 - pA) + pU(w_0 - L + A - pA). \tag{1}$$

- How much insurance would this person wish to buy (assuming they can buy up to their total wealth,  $w_0 - pL$ , at actuarially fair prices)? To solve for the optimal amount of insurance that

the consumer should purchase, maximize their utility with respect to the insurance policy:

$$\begin{aligned} \max_A E(U) &= (1-p)U(w_0 - pA) + pU(w_o - L + A - pA) \\ \frac{\partial E(U)}{\partial A} &= -p(1-p)U'(w_0 - pA) + p(1-p)U'(w_o - L + A - pA) = 0. \\ &\Rightarrow U'(w_0 - pA) = U'(w_o - L + A - pA), \\ &\Rightarrow A = L, \end{aligned}$$

which implies that wealth is  $w_0 - L$  in both states of the world (insurance claim or no claim).

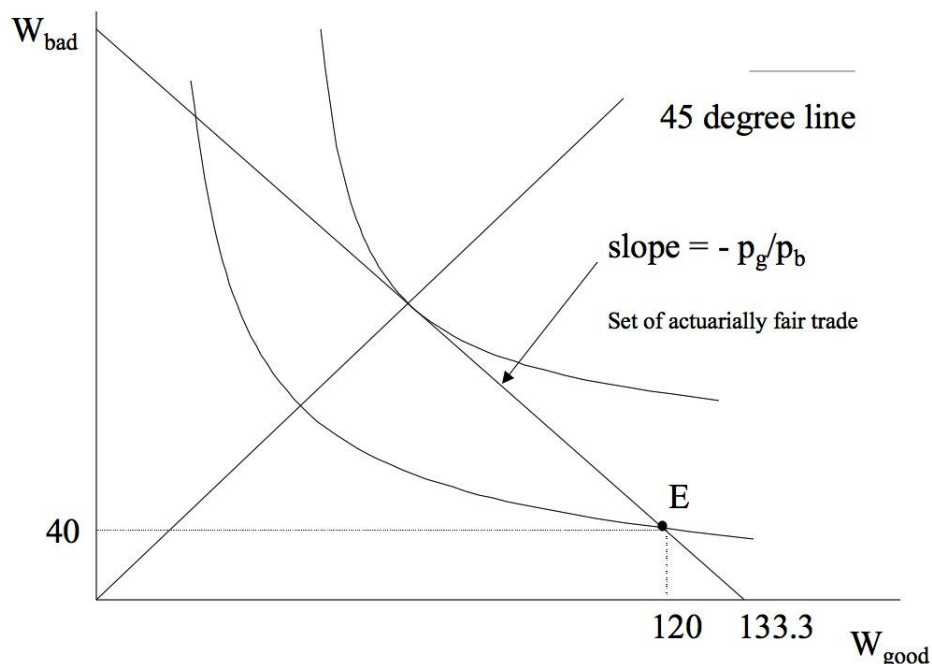
- A risk averse person will optimally buy *full insurance* if the insurance is actuarially fair.
- Is the person better off for buying this insurance? Absolutely. You can verify that expected utility rises with the purchase of insurance *although expected wealth is unchanged*.
- You could solve for *how much* the consumer would be willing to pay for a given insurance policy. Since insurance increases the consumer's welfare, s/he will be willing to pay some positive price *in excess of the actuarially fair premium* to defray risk.
- What is the intuition for why consumers want full insurance?
  - *The consumer is seeking to equate the marginal utility of wealth across states.*
  - Why? For a risk averse consumer, the utility of average wealth is greater than the average utility of wealth.
  - The consumer therefore wants to distribute wealth evenly across states of the world, rather than concentrate wealth in one state.
  - The consumer will attempt to maintain wealth at the same level in all states of the world, assuming she can costlessly transfer wealth between states of the world (which is what actuarially fair insurance allows the consumer to do).
  - This is exactly analogous to convex indifference curves over consumption bundles.
  - Diminishing marginal rate of substitution across goods (which comes from diminishing marginal utility of consumption) causes consumer's to want to diversify across goods rather than specialize in single goods.
  - Similarly, diminishing marginal utility of wealth causes consumers to wish to diversify wealth across possible states of the world rather than concentrate it in one state.
- Q: How would the answer to the insurance problem change if the consumer were *risk loving*?

- A: They would want to be at a corner solution where all risk is transferred to the least probable state of the world, again holding constant expected wealth.

## 2.1 Operation of insurance: State contingent commodities

- To see how risk preference generates demand for insurance, it is useful to think of insurance as a ‘state contingent commodity,’ a good that you buy in advance *but only consume* if a specific state of the world arises.
- Insurance is a state contingent commodity: when you buy insurance, you are buying a claim on \$1.00. This insurance is purchased before the state of the world is known. You can only make the claim for the payout if the relevant state arises. Thus, you pay the insurance company *regardless* of whether or not you make a claim. The insurance company pays you only if a bad outcome is realized (e.g., you have a car accident).
- Previously, we’ve drawn indifference maps across goods  $X, Y$ . Now we will draw indifference maps across states of the world: *Good, Bad*. You can equivalently think of Good and Bad as corresponding to no-accident and accident, respectively.
- Consumers can use their endowment (equivalent to budget set) to shift wealth across states of the world via insurance, just like the budget set can be used to shift consumption across goods  $X, Y$ .
- Example: Two states of world, good and bad, with  $w_0 = 120$ ,  $p = 0.25$ ,  $L = 80$ .

$$\begin{aligned}
 w_g &= 120 \\
 w_b &= 120 - 80 \\
 \Pr(g) &= (1 - p) = 0.75 \\
 \Pr(b) &= p = 0.25 \\
 E(w) &= 0.75(120) + .25(40) = 100 \\
 E(u(w)) &< u(E(w)) \text{ if consumer is risk averse.}
 \end{aligned}$$



- Let's say that this consumer can buy actuarially fair insurance. What will it sell for?

**Definition 1.** Actuarially fair insurance:

The price of the insurance policy exactly equals the expected monetary losses.

- If you want \$1.00 in Good state, this will sell at \$0.75 *prior to the state being revealed*. The reason is that the good state will occur with 75% probability.
- If you want \$1.00 in Bad state, this will sell for \$0.25 *prior to the state being revealed* because the bad state will occur with 25% probability.
- Note again that these prices reflect expected probabilities of making the claim. So, a *risk neutral* firm (say a central bank) could sell you insurance against bad states at a price of \$0.25 on the dollar and insurance against good states (assuming you wanted to buy it) at a price of \$0.75 on the dollar.
- The price ratio of payments in the Good state relative payments in the Bad state is therefore

$$\frac{P_g}{P_b} = \frac{p}{(1-p)} = 3.$$

- The set of fair trades among these states can be viewed as a 'budget set' and the slope of which is  $-\frac{P}{(1-P)}$ , and which passes through the initial endowment. Note that this slope is the relatively probabilities of two states.

- **Now we need indifference curves**

- Recall that the utility of this lottery (the endowment) is:

$$u(L) = pu(w_g) + (1 - p)u(w_b).$$

- Along an indifference curve

$$\begin{aligned} dU &= 0 = pu'(w_g)dw_g + (1 - p)u'(w_b)dw_b, \\ \frac{dw_b}{dw_g} &= -\frac{pu'(w_g)}{(1 - p)u'(w_b)} < 0. \end{aligned}$$

- Provided that  $u(\cdot)$  is concave, these indifference curves are bowed away from the origin in probability space. [It can readily be proven that indifference curves are convex to origin by taking second derivatives, but the intuition is straightforward.]
  - Flat indifference curves would indicate risk neutrality—because for risk neutral consumers, expected utility is linear in expected wealth.
  - Convex indifference curves mean that you must be compensated to bear risk.
  - i.e., if I gave you \$133.33 in good state and 0 in bad state, you are strictly worse off than getting \$100 in each state, even though your expected wealth is

$$E(w) = 0.75 \cdot 133.33 + 0.25 \cdot 0 = 100.$$

- So, I would need to give you more than \$133.33 in the good state to compensate for this risk.
  - Bearing risk is psychically costly, so the consumer must be compensated to hold the make them indifferent. (That is why the indifference curves are bowed away from the origin.)
  - Note that this implies that there are potential welfare improvements available from reducing risk if there were an inexpensive way to reduce it.
- In the figure above, the movement from the lower (closer to the origin) to the upper indifference curve is the gain from shedding risk.
  - Notice from the figure that, along the  $45^\circ$  line,  $w_g = w_b$ .

- But if  $w_g = w_b$ , this implies that

$$\frac{dw_b}{dw_g} = -\frac{pu'(w_g)}{(1-p)u'(w_b)} = \frac{p}{(1-p)} = \frac{P_g}{P_b}.$$

- Hence, the indifference curve will be tangent to the budget set at exactly the point where wealth is equated across states. [This is an alternative way of demonstrating the results above that a risk averse consumer will always fully insure if insurance is actuarially fair.]
- This is a very strong restriction that is imposed by the expected utility property: *The slope of the indifference curves in expected utility space must be tangent to the odds ratio.*

### 3 The Market for Insurance

Now consider how the market for insurance operates. If everyone is risk averse (and it's pretty safe to assume that most are), how can insurance exist at all? Who would sell it? There are actually three distinct mechanisms by which insurance can operate: risk pooling, risk spreading and risk transfer.

#### 3.1 Risk pooling

Risk pooling is the main mechanism underlying most *private* insurance markets. It applies the Law of Large Numbers to defray risk—which is to say that it makes risk disappear.

- As noted above, for any number of tosses  $n$  of a fair coin, the expected fraction of heads  $H$  is  $E(H) = \frac{0.5n}{n} = 0.5$ . But the variance around this expectation (equal to  $\frac{p(1-p)}{n}$ ) is declining in the number of tosses:

$$\begin{aligned} V(1) &= 0.25 \\ V(2) &= 0.125 \\ V(10) &= 0.025 \\ V(1,000) &= 0.00025 \end{aligned}$$

- We cannot predict the **share** of heads in one coin toss with any precision, but we can predict the **share** of heads in 10,000 coin tosses with considerable confidence. It will be vanishingly close to 0.5.
- Therefore, by *pooling* many independent risks, insurance companies can treat uncertain outcomes as *almost known*.

- So, “risk pooling” is a mechanism for providing insurance. It *defrays* the risk across independent events by exploiting the law of large numbers – making risk effectively disappear.

### 3.1.1 Example

- Let’s say that each year, there is a  $1/250$  chance that my house will burn down. If it does, I lose the entire \$250,000 house. The expected cost of a fire in my house each year is therefore about \$1,000.
- Given my risk aversion, it is costly in expected utility terms for me to bear this risk (i.e., much more costly than simply reducing my wealth by \$1,000).
- If 100,000 owners of \$250,000 homes all put \$1,000 into the pool, this pool will collect \$100 million.
- In expectation, 400 of us will lose our houses ( $\frac{100,000}{250} = 400$ ).
- The pool will therefore pay out approximately  $250,000 \cdot 400 = \$100$  million and approximately break even.
- Everyone who participated in this pool is better off to be relieved of the risk, though most will pay \$1,000 the insurance premium and not lose their house.
- However, there is still some risk that the pool will face a larger loss than the expected  $1/400$  of the insured.
- The law of large numbers says this variance gets vanishingly small if the pool is large and the risks are independent. How small?

$$V(Loss) = \frac{P_{Loss}(1 - P_{Loss})}{100,000} \frac{0.004(1 - 0.004)}{100000} = 3.984 \times 10^{-8}$$

$$SD(Loss) = \sqrt{3.984 \times 10^{-8}} = 0.0002$$

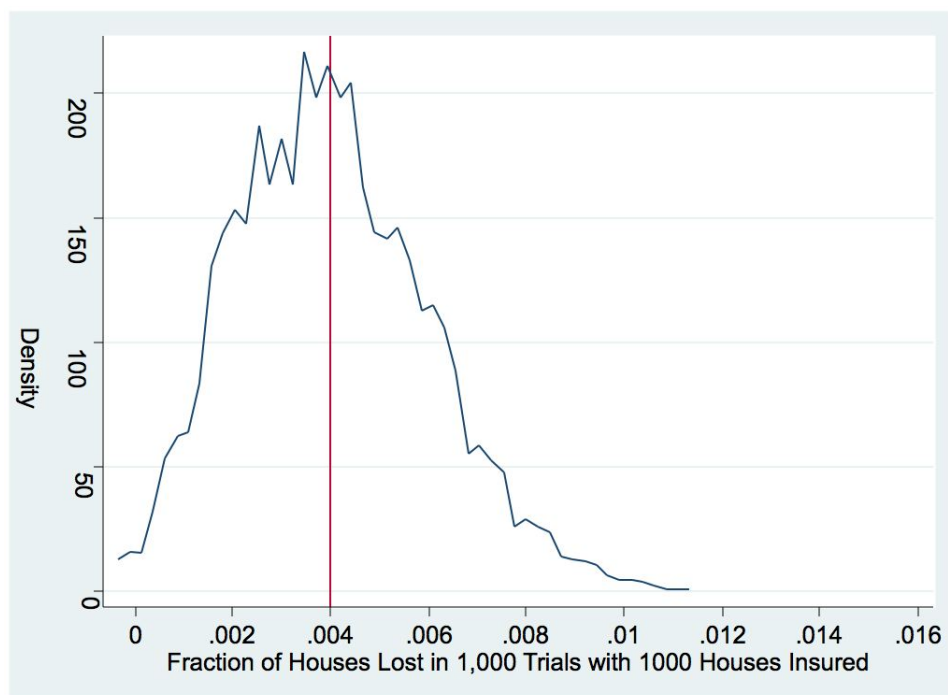
- Using the fact that the binomial distribution is approximately normally distributed when  $n$  is large, this implies that:

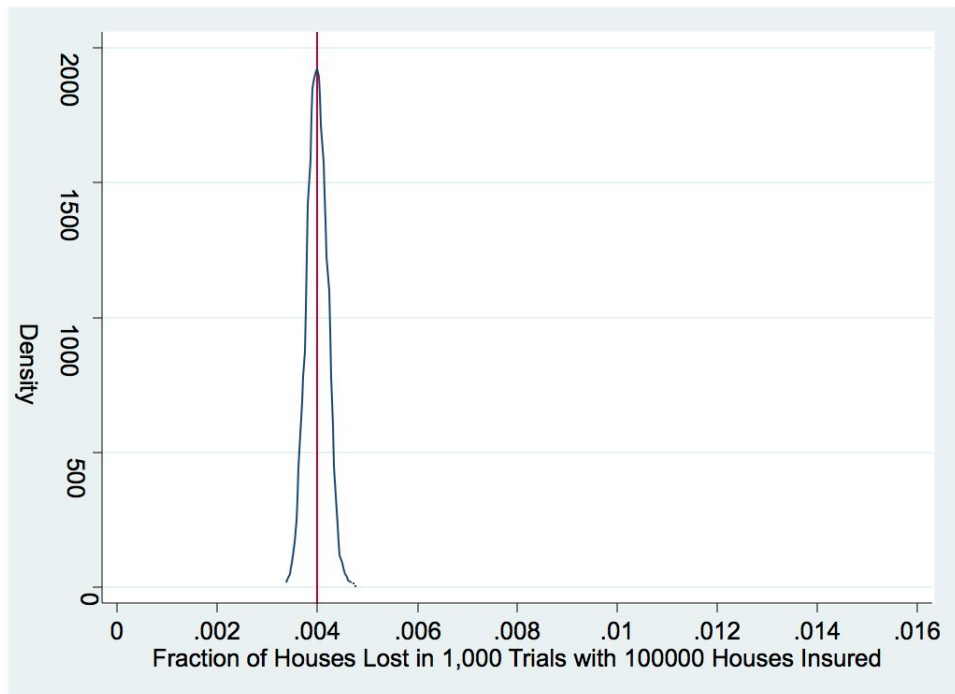
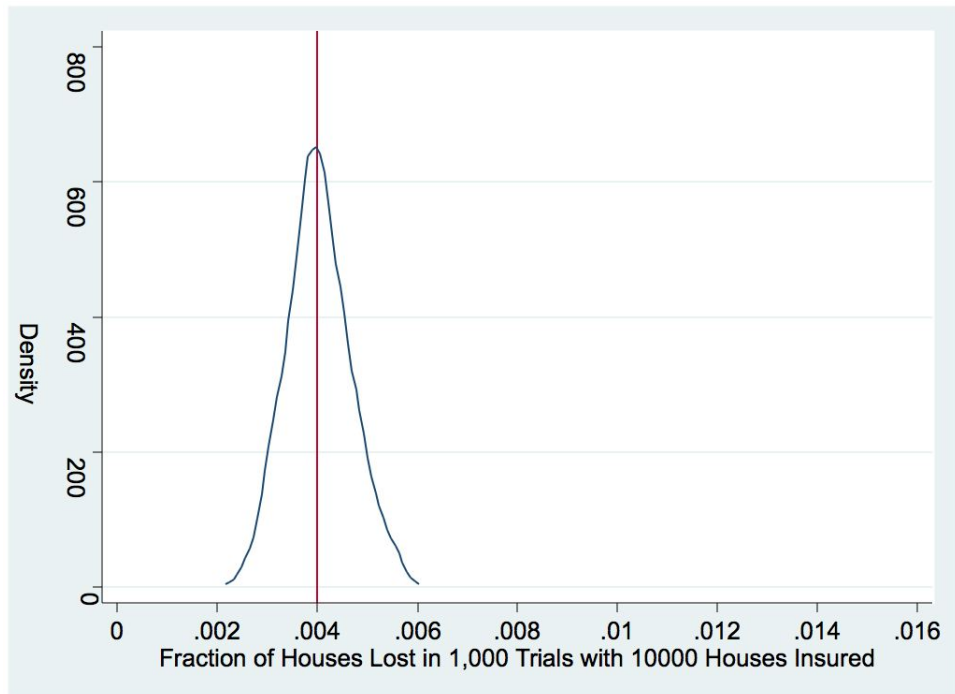
$$\Pr[Loss \in (0.004 \pm 1.96 \cdot 0.0002)] = 0.95$$

- So, there is a 95% chance that there will be somewhere between 361 and 439 losses, yielding a cost per policy holder in 95% of cases of \$924.50 to \$1,075.50.



- Most of the risk is defrayed in this pool of 100,000 policies.
- And as  $n \rightarrow \infty$ , this risk entirely vanishes.
- So, risk pooling generates a Pareto improvement (assuming we establish the insurance mechanism before we know whose house will burn down). Everyone in this example is better off buying the insurance.
- In class, I will also show a numerical example based on simulation. Here, I've drawn independent boolean variables, each with probability  $1/250$  of equalling one (representing a loss). I plot the frequency distribution of these draws for 1,000 replications, while varying the sample size (number of draws): 1,000, 10,000, 100,000, 1,000,000, and 10,000,000.
- This simulation shows that as the number of independent risks gets large (that is, the sample size grows), the odds that the number of losses will be more than a few percentage points from the mean contracts dramatically.
- With sample size 10,000,000, there is virtually no chance that the number of losses would exceed  $1/250 \cdot N$  by more than a few percent. Hence, pooling of independent risks effectively eliminates these risks – a Pareto improvement.





### 3.2 Risk spreading

- Q: When does this ‘pooling’ mechanism above *not* work? When risks are *not independent*. Possible examples:
  - Earthquakes

- Floods
- Epidemics
- When a catastrophic event is likely to affect many people simultaneously, it is (to some extent) **non-diversifiable**. This is why many catastrophes such as floods, nuclear war, etc., are specifically not covered by insurance policies.
- But does this mean there is no way to insured against these correlated risks?
- Actually, we can still ‘spread’ risk providing that there are some people likely to be unaffected.
- The basic idea here is that because of the concavity of the (risk averse) utility function, taking a little bit of money away from each person incurs lower social costs than taking a lot of money from a few people.
- Many risks cannot be covered by insurance companies, but the government can intercede by transferring money among parties. Many examples:
  - Victims compensation fund for World Trade Center.
  - Medicaid and other types of catastrophic health insurance.
  - All kinds of disaster relief.
- Many of these insurance “policies” are not even written until the disaster occurs—there was no market. But the government can still spread the risk to increase social welfare. Why is there no market? Because only those who are going to be affected are going to buy the insurance, meaning that the private market cannot spread the risk. However, the government can spread the risk, as people don’t have to “buy into” the government transfer.
- For example, imagine 10,000 people, each with VNM utility function  $U(w) = \ln(w)$  and wealth 100,000. Imagine that one of them experiences a loss of 50,000. His utility loss is

$$L = U(50,000) - U(100,000) = -0.693.$$

- Now, instead consider if we took this loss and distributed it over the entire population:

$$L = 10,000 \cdot [\ln(99,995) - \ln(100,000)] = -0.500.$$

The aggregate loss of  $-0.500$  is considerably smaller than the individual loss of  $-0.693$ . This comes from the concavity of the utility function.

- Hence, risk spreading may improve social welfare, even if it does not defray the total amount of risk faced by society.
- Does risk spreading offer a Pareto improvement? No, because we must take from some to give to others.

### 3.3 Risk transfer

- Third idea: if utility cost of risk is declining in wealth (constant absolute risk aversion for example implies declining relative risk aversion), this means that *less wealthy people could pay more wealthy people to bear their risks* and both parties would be better off.
- Again, take the case where  $U(w) = \ln(w)$ . Imagine that an individual faces a 50 percent chance of losing \$100,000. What would this person pay to eliminate this risk? It will depend on his or her initial wealth.
- Assume that initial wealth is 200,000. Hence, expected utility is

$$U(L) = 0.5 \ln 200,000 + 0.5 \ln 100,000 = 11.86$$

Expected wealth is \$150,000. The certainty equivalent of this lottery is  $\exp[11.86] = \$141,421$ . Hence, the consumer would be willing to pay up to \$8,579 to defray this risk. In other words, she would be indifferent between having \$141,421 for sure and a 50/50 chance of having either \$200K or \$100K.

- Now consider a person with the identical utility function and facing the same possibility of a \$100K loss but who is much wealthier. Say her wealth is 2,000,000. Expected utility is

$$u(L) = 0.5 \ln 2,000,000 + 0.5 \ln 1,900,000 = 14.45.$$

Expected wealth is \$1,950,000. The certainty equivalent of this lottery is  $\exp[14.45] = \$1,949,359$ . Hence, this consumer would be willing to pay only \$641 to avoid the lottery.

- The wealthy consumer could fully insure the poor consumer at psychic cost \$641 while the poor consumer would be willing to pay \$8,579 for this insurance. Any price that they can agree between (\$641, \$8,579) represents a pure Pareto improvement.
- Why does this form of risk transfer work? Because the logarithmic utility function exhibits declining absolute risk aversion—the wealthier someone is, the lower their psychic cost of bearing a fixed monetary amount of risk. Is this realistic? *Quite likely.* When you're a

teenager and \$20 falls out of a hole in your pocket and is gone for good, you feel completely crushed. When you're an adult and \$20 falls out of a hole in your Gucci briefcase, you think: "Hey, it's only a transfer. And that reminds me: I've been meaning to buy a new Gucci briefcase."

- Example: Lloyds of London used to perform this risk transfer role:
  - It took on large, idiosyncratic risks: satellite launches, oil tanker transport, the Titanic.
  - These risks were not diversifiable in any meaningful sense.
  - But companies and individuals are willing to pay a great deal to defray them.
  - Lloyds pooled the wealth of British nobility and gentry ('names') to create a super-rich consumer that in aggregate was much more risk tolerant than even the largest company.
  - For over a century, this idea generated large, steady inflows of cash for the 'names' that underwrote the Lloyds' policies.
  - Then Lloyds took on asbestos liability...
  - [For a fascinating account of how Lloyds bankrupted the British nobility, have a look at the 1993 *New Yorker* article by Julian Barnes, "The Deficit Millionaires." This article doesn't have much economic content, but it's gripping.]

## 4 Insurance markets: Conclusion

Insurance is potentially an extremely beneficial financial/economic institution, which can make people better off at low or even zero aggregate cost (in the case of risk pooling). We'll discuss shortly why insurance markets do not work as well in reality as they might in theory. But they still create enormous social value in the aggregate despite their imperfections.

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