

Fall 2018 14.01 Problem Set 4 - *Solutions*

Problem 1: True or False (24 points)

- (4 points) In the short and long run, a profit-maximizing firm will choose its input mix based on $MRTS = -\frac{w}{r}$.

Solution: False, in the short run the firm can't choose K , so this condition may not hold. This condition only holds in the long-run when the firm can choose optimally both capital and labor.
- (4 points) Long-run marginal costs can be lower or higher than short-run marginal costs, but long-run average costs can't be higher than the short-run average costs.

Solution: The long-run average costs can't be higher than the short-run average costs because in the long-run you can choose both inputs (L and K) while in the short-run you can only choose L . However, the marginal costs can be either higher or lower: think for example of a production function $F(L, K) = \sqrt{LK}$. The long-run marginal costs are $MC^{LR} = 2\sqrt{wr}$, while the short-run marginal costs are $MC^{SR} = \frac{2wq}{K}$, so for small q will have that $MC^{SR}(q) < MC^{LR}(q)$ but for large q we will have the opposite.
- (4 points) In a perfectly competitive market with identical firms, a permanent positive demand shock leads to a permanent increase in the price in the long run.

Solution: False, firm entry can bring the equilibrium price back to the same level as in the initial equilibrium. As seen in lecture, under some conditions the long run supply curve is perfectly elastic so the equilibrium price is equal to the minimum average total cost.
- (4 points) In a perfectly competitive industry, a profit-maximizing firm sets its price equal to its marginal cost in a range where the marginal cost is decreasing.

Solution: False, if the marginal cost is decreasing we would be at a local minimum.

5. (4 points) Adding up the individual supply curves $P = 5 + Q_1$ and $P = 3 + Q_2$ will lead to the market supply curve $P = 8 + 2Q$.

Solution: False, we have to add the curves horizontally, not vertically, it doesn't make sense to add prices. The market supply curve would be

$$Q(P) = Q_1(P) + Q_2(P) = \begin{cases} 2P - 8 & \text{if } P \geq 5 \\ P - 3 & \text{if } 3 \leq P < 5 \\ 0 & \text{if } P < 3 \end{cases}$$

6. (4 points) In 1998, the Kenyan government confiscated and burnt 12 tons of elephant ivory in a gesture to persuade the world to halt ivory trade. The equilibrium quantity in the market for ivory will surely decrease, while the effect on price is ambiguous: it may decrease if this gesture is effective in convincing consumers to stop buying ivory, and will increase otherwise.

Solution: True, both the supply and demand curves will shift left, so the quantity must go down. If the gesture is very effective, the demand will have a large shift inwards so the price will go down, while if it doesn't move much the price will increase.

Problem 2: Short-run and Long-run equilibrium (26 points)

Consider a market for skateboards that is in a long-run equilibrium. In this equilibrium, each firm's short-run and long-run total cost functions are given by:

$$\begin{aligned} SRTC(q) &= q^3 - 3q^2 + 3q + 4 \\ LRTC(q) &= 3q \end{aligned}$$

The market demand for skateboards is given by $Q_D(P) = 27 - P$.

1. (4 points) What is the equilibrium price in the initial long-run equilibrium?
Solution: Since the market is competitive, the equilibrium price must equal the minimum long-run average cost. Hence, $P^ = 3$.*

2. (4 points) Knowing that cost curves are defined by the above functions, explain why you can infer the number of skateboards each firm produces in long run equilibrium. Calculate the quantity. [Hint: If the market is in a long-run equilibrium, it is also in a short-run equilibrium.]

Solution: The short-run marginal cost has to equal the market price we found above. This gives $SRMC(q^) = P^* \Leftrightarrow 3q^{*2} - 6q^* + 3 = 3 \Leftrightarrow 3q^*(q^* - 2) = 0$ so we find that $q^* = 2$ (note that the solution $q = 0$ is a local minimum, so it is not optimal)*

3. (4 points) What is the equilibrium market quantity in the initial long-run equilibrium? How many firms are in the market?

Solution: Plugging the equilibrium price into the demand curve, we get $Q^ = 24$. Furthermore, since every firm produces 2 units, this means that there have to be 12 firms.*

4. (6 points) Derive each firm's short-run supply curve (expressing it as q as a function of p). Derive the short-run market supply curve.

Solution: Remember that the short-run supply curve is the marginal cost curve ($P = MC(q)$), in the range where $P \geq AVC(q)$; otherwise, the firm prefers to shut down. Solving $P = MC(q)$ for q gives:

$$\begin{aligned} P &= 3q^{*2} - 6q^* + 3 \\ \Rightarrow q &= 1 + \sqrt{\frac{P}{3}} \end{aligned}$$

However, we need that $P = MC(q) \geq AVC(q) \Leftrightarrow q \geq 3/2 \Leftrightarrow P \geq \frac{3}{4}$. If $P < 3/4$, supply is 0. So the individual supply curve is

$$q(p) = \begin{cases} 1 + \sqrt{\frac{P}{3}} & \text{if } p \geq \frac{3}{4} \\ 0 & \text{if } p < \frac{3}{4} \end{cases}$$

This means the short-run market supply is

$$Q(p) = \begin{cases} 12 + 12\sqrt{\frac{P}{3}} & \text{if } p \geq \frac{3}{4} \\ 0 & \text{if } p < \frac{3}{4} \end{cases}$$

5. (4 points) The skateboards suddenly come into vogue, and the market demand shifts to $Q'_D(P) = 57 - 3P$. What are the equilibrium price and quantity in the short run?

Solution: We need to find the intersection between the market supply curve that we found before and the new demand curve: $12 + 4\sqrt{3P} = 57 - 3P \Rightarrow P = \frac{25}{3}$. This implies that $Q = 32$.

6. (4 points) If the market demand stays at Q'_D thereafter, how will the market adjust? How many firms will there be in the long run?

Solution: Because nothing has changed on the supply side, the new entering firms would drive the price back down to $P = 3$ and to find the quantity we just have to evaluate the new demand at the equilibrium price $P = 3$, so we obtain a quantity of $Q = 48$. Since each firm produces 2 units at this price, there have to be 24 firms.

Problem 3: Long-run equilibrium with heterogeneous firms (34 points)

Consider the market for bicycles. There are two technologies used by firms in this industry: Technology 1 uses solar power, and has a cost function $C^1(q) = q + 4q^2 + 32$ for $q > 0$. Technology 2 uses electricity from the grid and is more efficient, with a cost function $C^2(q) = q + 2q^2 + 32$ for $q > 0$. Assume that we are in the long run, so firms using both technologies can shut down and leave the market at 0 cost, so that $C(0) = 0$ for both technologies.

1. (4 points) What are the marginal and average cost curves for each of these two technologies? In the long-run, assuming that firms can choose their technology, will any firms choose the solar technology (technology 1)? Why or why not?

Solution: The MC and AC curves are (denoting the technology type with superscript 1 and 2 respectively):

$$\begin{aligned}MC^1(q) &= 1 + 8q \\MC^2(q) &= 1 + 4q \\AC^1(q) &= 1 + 4q + \frac{32}{q} \\AC^2(q) &= 1 + 2q + \frac{32}{q}\end{aligned}$$

No firms will choose the solar technology. Looking at the two cost functions, we can see that $AC^1(q) > AC^2(q) \forall q$. Therefore, firms will choose technology 2 regardless of their desired level of output.

2. (6 points) Find the individual supply curve of a firm operating Technology 2.

Solution: Given price p , firms will choose q to maximize profits. They will do so by setting $p = MC$. For technology 2 this means $p = 1 + 4q \Rightarrow q_2(p) = \frac{p-1}{4}$. We also have to take into account that the firm can choose to exit the market (i.e. produce $q = 0$), so the supply curve will coincide with the marginal cost curve only when it is above the average cost curve. Therefore, we get

$$q_2(p) = \begin{cases} \frac{p-1}{4} & \text{if } p \geq 17 \\ 0 & \text{if } p < 17 \end{cases}$$

3. (4 points) Suppose that market demand for bicycles is given by $D(p) = 820 - 40p$. What will be the long-run price in the market? How much will each firm produce at this price? What will the total number of firms be?

Solution: With free entry, we know that firms will continue entering the market until profits are 0. Therefore, in the long-run equilibrium the price must be equal to the minimum average total cost, so $P^ = 17$. To find the equilibrium quantity*

we evaluate the demand function at this price, so we obtain $Q = 140$. To find the quantity produced by each individual firm, we evaluate the individual supply curve at the equilibrium price, so we obtain $q = 4$. Because each firm produces $q = 4$, the total number of firms, $N = \frac{140}{4} = 35$.

4. (6 points) Now, suppose that the government offers solar subsidies to 10 bicycle manufacturers. These subsidies are for \$28 and the manufacturers receive these subsidies as long as they produce a positive quantity of bikes with the solar technology (i.e. technology 1). What are new AC, MC, and supply curve for the solar technology with the subsidy?

Solution: The MC, AC, and supply curves for technology 1 (the solar technology) with the subsidy are:

$$\begin{aligned} MC^1(q) &= 1 + 8q \\ AC^1(q) &= 1 + 4q + \frac{4}{q} \\ q^1(p) &= \begin{cases} \frac{p-1}{8} & \text{if } p \geq 9 \\ 0 & \text{if } p < 9 \end{cases} \end{aligned}$$

5. (6 points) What will be the long run price now that there are the 10 bicycle manufacturers using technology 1 (assuming that there is still free entry for firms using technology 2)? What quantity will be produced by firms using technology 1 and 2? In equilibrium, how many firms using technology 2 will there be in the market?

Solution: At $p = 17$, the firms with technology 1 will each supply $q^1(p) = \frac{17-1}{8} = 2$. Consequently, in total technology 1 firms will supply 20 bicycles, leaving technology 2 firms to supply the remaining 120 bicycles demanded when $p = 17$. Because the price is still 17, we know that each technology 2 firm will still produce 4 bicycles. Consequently, the total number of technology two firms will be $N^2 = \frac{120}{4} = 30$.

6. (4 points) Will either type of firm make any profit in equilibrium? If so, how much will they make? If your results differ by firm, explain the intuition for why firms using some technologies make profits while others do not.

Solution: We know from above that at $p^ = 17$, technology type 2 firms make zero profits. However, type 1 firms do make profits. To see this, note that $\pi^1(p) = pq(p) - q(p) - 4q(p)^2 - 4$. Plugging in equilibrium prices and quantities yields $\pi^1(p) = 34 - 2 - 16 - 4 = 12$. Therefore, type 1 firms do make positive profits. Type 1 firms make profits because there are barriers to entry using their production technology: the government only issues subsidies to 10 firms. Conversely, there is free entry of technology 2 firms, so they will enter until their profits are driven down to 0.*

7. (4 points) Now suppose that the government increases the number of solar bike manufacturing subsidies it will give from 10 to 500. What is the new long-run

price? How much will be produced by firms of each type? How many firms will their be of each type? Do any firms make profits?

Solution: At a price $p = 9$, we would have that firms using the solar technology are indifferent between staying out of the market or producing $q = 1$ (either way, they get zero profits). So at price $p = 9$ the supply can be anything between 0 and 500. Meanwhile, at this price demand will be 460, so we have that in equilibrium we will have 460 firms using the solar technology supplying 1 unit each. No firms with the other technology will participate in the market, and we have that all firms make zero profits.

Problem 4: Consumer and Producer Surplus (16 points)

Suppose the demand for apples is $Q^D = 550 - 50P$ and the industry supply curve is $Q^S = -12.5 + 62.5P$.

1. (4 points) Calculate the equilibrium price and quantity.

Solution:

$$550 - 50P = -12.5 + 62.5P$$

$$562.5 = 112.5P$$

$$P^* = 5$$

$$Q^* = 300$$

2. (6 points) Compute the consumer, producer, and total surplus for this market.

Solution:

$$CS = \frac{1}{2} (11 - 5) 300 = 900$$

$$PS = \frac{1}{2} (5 - 0.2) 300 = 720$$

3. (6 points) Suppose that the government gives producers a subsidy of \$2 per bushel of apples sold. Draw the effect on the demand and supply curves, with quantity on the horizontal axis and the price paid by consumers on the vertical axis. Compute the new equilibrium price and quantity, the consumer and producer surplus, and the government expenditure on the subsidy. Compare the government expenditure with the increase in consumer and producer surplus.

Solution: When consumers pay a price p , producers receive $p + 2$. Therefore, the

new equilibrium will be such that

$$\begin{aligned}
 Q^D(p) &= Q^S(p+2) \\
 550 - 50p &= -12.5 + 62.5(p+2) \\
 437.5 &= 112.5p \\
 p &= 3.89 \\
 Q &= 355.55
 \end{aligned}$$

Since consumers are paying a lower price, they will get a higher consumer surplus

$$CS = \frac{1}{2} (11 - 3.89) 355.55 = 1263.98$$

Producer surplus will also be higher because firms are receiving a higher price

$$PS = \frac{1}{2} (5.89 - 0.2) 355.55 = 1011.54$$

The total increase in producer and consumer surplus is

$$\Delta CS + \Delta PS = (1263.98 - 900) + (1011.54 - 720) = 655.52$$

Meanwhile, the government expenditure on the subsidy program is

$$Gov\ Expenditure = 2 \times 355.55 = 711.1$$

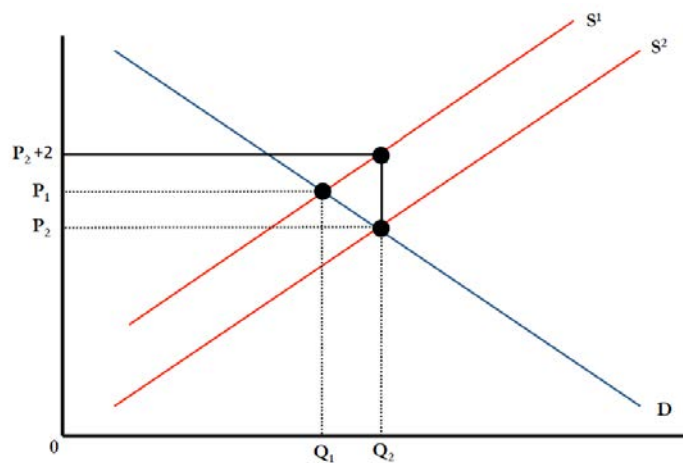


Figure 1:

MIT OpenCourseWare
<https://ocw.mit.edu>

14.01 Principles of Microeconomics
Fall 2018

For information about citing these materials or our Terms of Use, visit: <https://ocw.mit.edu/terms>.