Fall 2018 14.01 Problem Set 10

Problem 1: True or False (12 points)

Determine whether the following statements are True or False. Explain your answer.

1. *(3 points)* Consider a gamble that pays $10 with probability 50% and $0 with probability 50%. No expected-utility maximizing individual will strictly prefer this gamble over a certain payment of 5.

2. *(3 points)* Consider a gamble with which you gain $50 with probability 20%, you gain $130 with probability 30%, or you lose $95 with probability 50%. A risk-neutral individual would take this gamble.

3. *(3 points)* Consider an economy with two people, Alan and Brian, and a single good: potatoes. Both Alan and Brian share the same utility function $u = \sqrt{p}$, where p is the number of potatoes they possess. Now consider a following transfer: Alan gives some potatoes to Brian, but half of which are lost in transportation. A government who maximizes the utilitarian social welfare function will never prefer such a transfer.

4. *(3 points)* A country that is a net exporter of a good, in the absence of any government subsidies or interventions, must always have advanced technology in producing that good

Problem 2: Diversification (30 Points)

Oliver has an endowment of $10,000 that he wants to invest. He can either invest in

- a bond, which yields 1%,

- the stock market, which consist of two firms, Amazon and Toys R Us

Each firm's stock is costs \$100 today, and will be worth \$400 in one year with probability $\frac{1}{2}$ or will drop to \$0 with probability $\frac{1}{2}$. Assume that the evolution of both stocks is independent: that is, the probability that stock of Amazon rises in value does not vary or depend on what has happened to stock of Toys R Us, and vice-versa.

Finally, assume that Oliver's utility function is $U(w) = \sqrt{w}$, and there is no inflation.

1. *(5 points)* Suppose that due to institutional regulations, Oliver can invest only in bonds, or only in Amazon. He cannot buy Toys R Us stock and he cannot buy both Amazon stock and bonds. What is Oliver's utility of buying bonds? What if he invests only in Amazon's stock? What does he prefer?

2. *(10 points)* Now suppose there is a change in regulations. Oliver can invest in either the stock market or in bonds, but not both. If Oliver decides to invest in the stock market, he can choose how much he wants to invest in each company. Let x denote the fraction of wealth that Oliver puts into Amazon and $1 - x$ denote the fraction of wealth Oliver puts into Toys R Us. If Oliver maximizes his expected utility, what should be the value of x?

3. *(5 points)* What is the expected value of the strategy in the previous question, and Oliver's expected utility? Does Oliver prefer to use this strategy or to invest in bonds? What is the effect at work here? (3 points)

4. *(5 points)* Suppose again that Oliver can only invest in stocks or only invest in bonds. Additionally, now Toys R Us has gone bankrupt, but Oliver could invest in a third company, Walmart, whose stock today is worth \$100. Interestingly, the value of the stocks for Amazon and Walmart is negatively correlated, so that with probability 1/2 Amazon goes bankrupt and Walmart's stocks are worth \$400, and with probability 1/2 the opposite happens. What is Oliver's optimal investment? Is correlation hurting or helping Oliver?

5. *(5 points)* Suppose Oliver works for Amazon and he is investing for his retirement. Is the investment decision you found in last question still optimal? Why, or why not?

Problem 3: European Potatoes (subsidy vs. welfare) (25 points)

Assume that the EU subsidizes the exports of potatoes to support employment in agriculture.

Supply and demand of potatoes of European producers and consumers are equal to $p = 400 + 10q_s$ and $p = 1000 - 2q_d$ respectively (where prices are in dollars and quantities in tons).

1. *(3 points)* First consider the case of Europe under autarky. What is the equilibrium price and quantity of potatoes when there is no subsidy?

2. *(3 points)* Assume that under free trade, the global price is $800 and European consumers can import however many tons of potatoes at that price per ton. Does Europe import or export potatoes at this price? How much does the EU import or export?

3. *(5 points)* Now, to support employment in agriculture, the EU starts subsidizing potatoes by 6.25% such that the price European producers get from selling potatoes rises to 850 (consumers are still paying the old price, but the government gives the producers an additional 50 dollars per ton of potato). How does this change the supply of potatoes for European producers? What is the new equilibrium and how does the subsidy affect potato exports or imports?

4. *(4 points)* How much does EU consumer surplus change?

5. *(5 points)* How much does EU producer surplus change?

6. *(5 points)* How much does this subsidy program cost the EU ? What is the change in total surplus from this program? Explain why total surplus decreases

Problem 4: Taxes and Redistribution (33 points)

Consider an economy with two types of individuals: the skilled and the unskilled. The only difference between the two is that the skilled have a higher wage $w_s = 15$ than the unskilled do, $w_u = 5$. Suppose 1/7 of the population is skilled and 6/7 are unskilled. Suppose that each individual has a utility function over consumption (c) and labor (L) of the following form:

$$U(c, L) = c - \frac{1}{2}L^2$$

where $0 \leq L \leq 24$ Each individual's income is then $w_j L$, where $j = s, u$.

1. (a) *(3 points)* Write down the individual's budget constraint in terms of consumption and *leisure* . Draw the different budget constraints for the skilled and unskilled

   (b) *(4 points)* Solve for each individual's optimal leisure, labor, and consumption choice.

2. Now suppose the government decides to levy an income tax. This tax collects $\frac{1}{5}$ of each individual's earnings. *For the moment* , assume it takes this tax but doesn't do anything with it.

   (a) *(3 points)* On the same set of axes, draw the *new* budget constraints faced by the two types of individuals. Again, label them clearly, with leisure on the x-axis.

   (b) *(4 points)* Solve for each individual's new optimal leisure, labor, and consumption.

   (c) *(4 points)* Compute the total *per-capita* tax collected by the government.

3. Now suppose that the government uses the tax revenue to give an equal amount T to each resident at the beginning (this is called a lump-sum transfer).

   (a) *(4 points)* Show that the lump sum transfer T will *not* affect the optimal labor supply of either type of individual.

   (b) *(6 points)* Suppose the government sets T so that it spends what it collects (the government balances its budget). Show that this tax-plus-transfer policy is good for the unskilled and bad for the skilled (relative to the no-tax regime from part 1).

4. *(3 points)* Now suppose the government wants to maximize a *utilitarian* social welfare function. Would it prefer setting no taxes, as in part 1 of this section, or would it prefer to set an income tax of $\frac{1}{5}$ and redistribute it through an equal lump-sum transfer T, as in part 3 of this question?

5. *(2 points)* Which tax scheme would a Rawlsian social planner prefer? (No taxes vs. setting an income tax of 20% and redistributing this wealth equally).

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