Fall 2018 14.01 Problem Set 9 - *Solutions*

Problem 1: True or False (20 points)

Determine whether the following statements are True or False. Explain your answer.

1. *(5 points)* Suppose the cost of making a car is cheapest in Japan. Then Japan should specialize in producing cars.
   Solution: False. If Japan can make cars more cheaply than any other country in the world, then Japan has an absolute advantage in making cars. However, gains from trade happen from countries specializing in their areas of comparative advantage, and absolute advantage does not imply comparative advantage

2. *(5 points)* In order for an IRA to encourage savings (relative to a regular savings account), the substitution effect of higher interest rates must dominate the income effect.
   Solution: True. IRAs offer higher returns to savings relative to regular savings account. For higher interest rates to increase savings, the substitution effect must dominate the income effect.

3. *(5 points)* An index fund is a portfolio of stocks that tracks a broader index such as the Dow Jones Industrial Average or the S&P 500. Investing in an index fund is better than investing in the stock of an individual company because the index fund always has higher returns.
   Solution: False. There is no guarantee that an index fund offers higher returns than the individual stock of a company (consider Amazon stock vs. the DJIA over the past 15 years). However, index funds offer the benefit of diversification, which individual stocks do not.

4. Consider the effect of interest rates on consumption today. Increasing interest rates always has a negative substitution effect (decreases consumption today) and a positive income effect (increases consumption today)
   Solution: False. The income effect is ambiguous and depends on whether the person is a borrower or a saver.

Problem 2: Trade and Production Possibilities Frontier (20 points)

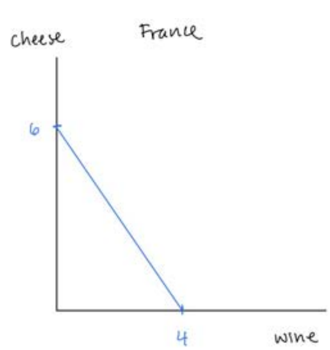
Consider the production of wine and cheese in France and Spain. This table gives the number of necessary hours to produce each (labor is the only input):

	France	Spain
1 Kilo of Cheese	4	6
1 Bottle of wine	6	12

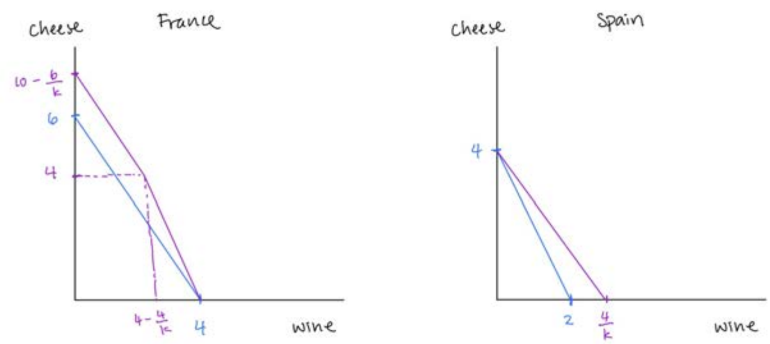
1. *(5 points)* For each good, which country has an absolute advantage? For each good, which country has a comparative advantage? (5 points)
   Solution: France has an absolute advantage in both Cheese and wine, because it takes fewer hours for France to produce each good.
   Spain has a comparative advantage in Cheese, with 1 Kilo of Cheese costs 6/12=0.5 a Bottle of wine while in France 1 Kilo of Cheese costs 4/6=0.67 a Bottle of wine. Therefore the France has a comparative advantage in wine; in France a bottle of wine costs 6/4=1.5 kilos of Cheese, while in the Spain it costs 12/6=2 kilos of cheese.

2. *(5 points)* Is it in Spain's interest to develop trade relationships with France? Is it in France's interest to trade with Spain? Is France more competitive at producing both goods?
   Solution: France does have an absolute advantage, but it will be in Spain's interest to trade because specialization will increase the values of production and of consumption in Spain. It is also in France's interest to trade with Spain, since Spain has a comparative advantage in producing cheese

3. *(5 points)* Suppose that France and Spain are under autarky (no trade). Draw the production possibility frontier for each country for the number of goods they can produce in one day (24 hours, one worker).
   Solution:



4. *(5 points)* France and Spain decide to trade and suppose they agree to trade one bottle of wine for k kilos of cheese. What values of k would make both France and Spain strictly better off under trade? Draw the new consumption set for each country under trade. How has it changed and why?
Solution: Under autarky, France can exchange one bottle of wine for $\frac{3}{2}$ *kilo of cheese. Spain can exchange one bottle of wine for 2 kilos of cheese. Therefore, any value of* $k \in (3/2, 2)$ *would make both Spain and France better off.*



(Note to graders: do not take off points if the consumption set for France is drawn as a straight line between $4k$ *on the cheese axis and* 4 *on the wine axis).*
Since Spain does not have an absolute advantage in anything, it will choose to specialize in making the product in which it has comparative advantage (cheese) so the consumption set is a straight line between 4 kilos of cheese and $4/k$ *bottles of wine.*
If France decides to consume less than 4 kilos of cheese (per day), then it should import the cheese from Spain. If France wants to consume more than 4 kilos of cheese, it will import the first 4 kilos from Spain, and produce the rest of the cheese itself. To obtain 4 kilos of cheese from spain, France needs to give spain $4/k$ *bottles of wine. We can see that under trade, both Spain and France can choose consumption bundles that were not feasible under no-trade.*

Problem 3: International Trade I : Differences in Technology (15 points)

Suppose there are only two goods in the world: tea and coffee. In both the US one pound of tea requires 3 hours of labor to produce and one pound of coffee requires 2 hours of labor to produce. A worker can choose to work either in the tea industry or in the coffee industry (skills are completely transferable across industries) and consider the case when the labor market is perfectly competitive, and the market for tea and coffee are also perfectly competitive.

1. *(5 points)* What is the ratio of the price of tea to the price of coffee?
Solution: Since the labor market is competitive, the wage per hour of labor must be the same in the two industries. Since the goods market is also competitive, prices in the tea industry satisfy

$$p_t MPL = w \implies \frac{1}{3}p_t = w \implies p_t = 3w$$

and prices in the coffee industry satisfy

$$p_c MPL = w \implies \frac{1}{2}p_c = w \implies p_c = 2w$$

Therefore, the ratio of the price of a pound of tea to a pound of coffee is $3/2$.

2. *(5 points)* Suppose that on the international market, due to the different production functions by different countries, we can trade k pounds of tea for 1 pound of coffee. For what values of k will the US choose to export tea? For what values of k will the US choose to export coffee
Solution: The US will choose to export the product it has a comparative advantage in producing and import the other product. Under autarky, one pound of coffee costs $2/3$ *one pound of tea. Therefore, for* $k > \frac{2}{3}$ *the US will choose to export coffee. For* $k < \frac{2}{3}$ *the US will choose to export tea.*

3. *(5 points)* For what values of k will the US be strictly better off under trade than under autarky? Why?
Solution: As long as $k \neq \frac{2}{3}$*, the US will be better off with trade. This is because when* $k \neq \frac{2}{3}$*, the US will have strict comparative advantage in producing one of the goods and will specialize in that good.*

Problem 4: International Trade II: Differences in Factor Abundance (45 points)

Suppose that there are only two products, computers and automobiles. Automobiles are more labor-intensive (requires relatively more labor) and are produced according to the production function

$$F^A(K, L) = K^{\frac{1}{3}}L^{\frac{2}{3}}.$$

Computers are more capital intensive (require relatively more capital) and are produced according to the function

$$F^C(K, L) = K^{\frac{2}{3}}L^{\frac{1}{3}}$$

Suppose that both labor and capital are perfectly mobile across the two industries. That is, workers and capital can switch fluidly from producing computers to producing cars and vice versa. For this problem, assume everything is in perfect competition.

1. *(5 points)* If labor and capital are perfectly mobile across the two industries, what must be true about wages and the price of capital in the two industries? That is, if w_A and w_C are the wages per unit of labor paid to workers in the automobile and computer industries respectively, what can we say about w_A and w_C? Likewise, if r_A and r_C are the price of capital in the two industries, what must be true about r_A and r_C?
   Solution: Since factors are perfectly mobile, $w_A = w_C$ and $r_A = r_C$. For the rest of the problem, we use w to refer to the wage and r to denote the price of capital.

2. *(10 points)* Suppose that the United States has 30 units of labor total and 240 units of capital, and all labor and capital is utilized for production. The price of automobiles is $p_A = 200$ while the price of computers are $p_c = 100$ under autarky. How much labor and capital is used for the production of automobiles under autarky? How much labor and capital is used to produce computers under autarky? *(Hint: Use the condition on wages and prices of capital you found in the previous exercise. In perfectly competitive labor and capital markets, what is the relationship between wages (or price of capital), the production function, and the price of the output goods?)*

   Solution: Let K_A denote the total amount of capital used to produce automobiles and and L_A denote the total number of workers used to produce automobiles. For wages to be the same in the two industries, we must have

$$
\begin{aligned}
200 \cdot MPL_A &= w = 100 \cdot MPL_C \\
\implies \frac{400}{3}\left(\frac{K_A}{L_A}\right)^{\frac{1}{3}} &= \frac{100}{3}\left(\frac{240-K_A}{30-L_A}\right)^{\frac{2}{3}} \\
\implies 64\left(\frac{K_A}{L_A}\right) &= \left(\frac{240-K_A}{30-L_A}\right)^{2}
\end{aligned}
$$

   Likewise,

$$
\begin{aligned}
200 \cdot MPK_A &= r = 100 \cdot MPK_C \\
\implies \frac{200}{3}\left(\frac{K_A}{L_A}\right)^{\frac{-2}{3}} &= \frac{200}{3}\left(\frac{240-K_A}{30-L_A}\right)^{\frac{-1}{3}} \\
\implies \left(\frac{K_A}{L_A}\right)^{-2} &= \left(\frac{240-K_A}{30-L_A}\right)^{-1}
\end{aligned}
$$

   This gives us two equations in two unknowns, which we can solve to get

$$
K_A = 80, \quad L_A = 20, \quad K_C = 240 - K_A = 160, \quad L_C = 30 - 20 = 10
$$

3. *(2 points)* How many cars and computers does the US produce?
   Solution: The US produces 31.748 *automobiles and* 63.496 *computers*

4. *(3 points)* What is the wage and price of capital in the US under autarky?
   Solution:
$$w = \frac{400}{3}\left(\frac{K_A}{L_A}\right)^{\frac{1}{3}} = \frac{400}{3}\left(\frac{80}{20}\right)^{\frac{1}{3}} \approx 211.7$$
$$r = \frac{200}{3}\left(\frac{K_A}{L_A}\right)^{\frac{-2}{3}} = \frac{200}{3}\left(\frac{80}{20}\right)^{\frac{-2}{3}} \approx 26.9987$$

5. *(5 points)* Assume that China has the same production function, but the price of automobiles is 100 and the price of computers is 200. China has 240 units of labor available and 30 units of capital. If all available labor and capital are used in production, how much labor and capital is used in each sector?
   Solution: Shortcut: notice that capital is to China what labor is to the US. That is, China and the US are perfectly symmetric, with the role of labor and capital flipped ,and the role of the two industries flipped. So we can take all the answers for the US, but change capital to labor and vice versa, doing the same for the industries.

   Long way:
   Let K_A denote the total amount of capital used to produce automobiles and and L_A denote the total number of workers used to produce automobiles. For wages to be the same in the two industries, we must have
$$100 \cdot MPL_A = w = 200 \cdot MPL_C$$
$$\implies \frac{100}{3}\left(\frac{K_A}{L_A}\right)^{\frac{1}{3}} = \frac{400}{3}\left(\frac{240-K_A}{30-L_A}\right)^{\frac{2}{3}}$$
$$\implies \left(\frac{K_A}{L_A}\right) = 64\left(\frac{240-K_A}{30-L_A}\right)^{2}$$

   Likewise,
$$100 \cdot MPK_A = r = 200 \cdot MPK_C$$
$$\implies \frac{200}{3}\left(\frac{K_A}{L_A}\right)^{\frac{-2}{3}} = \frac{200}{3}\left(\frac{240-K_A}{30-L_A}\right)^{\frac{-1}{3}}$$
$$\implies \left(\frac{K_A}{L_A}\right)^{-2} = \left(\frac{240-K_A}{30-L_A}\right)^{-1}$$

This gives us two equations in two unknowns, which we can solve to get

$$K_A = 10, \quad L_A = 160, \quad K_C = 30 - K_A = 20, \quad L_C = 240 - L_A = 80$$

6. *(2 points)* How many cars and computers does China produce?
Solution: China produces 31.748 *computers and* 63.496 *automobiles*

7. *(3 points)* What is the wage and price of capital in China under autarky?
Solution:

$$w = \frac{200}{3}\left(\frac{K_A}{L_A}\right)^{\frac{1}{3}} = \frac{200}{3}\left(\frac{10}{160}\right)^{\frac{1}{3}} \approx 26.4567$$

$$r = \frac{100}{3}\left(\frac{K_A}{L_A}\right)^{\frac{-2}{3}} = \frac{100}{3}\left(\frac{10}{160}\right)^{\frac{-2}{3}} \approx 211.653$$

8. *(5 points)* Why are wages and the price of capital different across the US and China?
Solution: There are two reasons for the differences in wages and price of capital. First, the labor is more abundant in China whereas in the US, labor is relatively scarce. As a result, wages are lower in China than in the US. The opposite is true for capital. Additionally, the price of the goods are different across the two countries.

9. *(10 points)* Suppose that the two countries open up to trade and the new equilibrium world price of automobiles is 150 and the world price for computers is also 150.

   (a) Is it still the case that both countries produce positive amounts of both goods? *(Hint: The condition you found in part 1 of this problem assumes that the countries produce a positive amount of both goods. Assume this condition holds and solve for the optimal capital and labor in each sector in US and China. Does this lead to any contradictions or impossibilities?)*
   Solution: We first assume that the US and China still produce positive amounts of both computers and automobiles. In the US, we get

$$150 \cdot MPL_A = w = 150 \cdot MPL_C$$

$$\Longrightarrow \frac{300}{3}\left(\frac{K_A}{L_A}\right)^{\frac{1}{3}} = \frac{150}{3}\left(\frac{240 - K_A}{30 - L_A}\right)^{\frac{2}{3}}$$

$$\Longrightarrow 8\left(\frac{K_A}{L_A}\right) = \left(\frac{240 - K_A}{30 - L_A}\right)^2$$

Likewise,

$$
\begin{aligned}
150 \cdot MPK_A &= r = 150 \cdot MPK_C \\
\implies \frac{150}{3}\left(\frac{K_A}{L_A}\right)^{\frac{-2}{3}} &= \frac{300}{3}\left(\frac{240-K_A}{30-L_A}\right)^{\frac{-1}{3}} \\
\implies \left(\frac{K_A}{L_A}\right)^{-2} &= 8\left(\frac{240-K_A}{30-L_A}\right)^{-1}
\end{aligned}
$$

Solving this yields $K_A = -60, L_A = -120$*, which cannot happen. This implies that the US will specialize in producing computers. Since the US and China are symmetric, China will specialize in producing automobiles.*

(b) What happens to the wages and the price of capital in the two countries? What happens to the total units produced in the two countries? Explain why this has happened.
Solution: The wage and price of capital in the US is then given by

$$
w_{USA} = \frac{150}{3}\left(\frac{240}{30}\right)^{\frac{2}{3}} = 200, \quad r_{USA} = \frac{300}{3}\left(\frac{240-K_A}{30-L_A}\right)^{\frac{-1}{3}} = 50
$$

while in China, the wage and the price of capital are given by

$$
r_{CHN} = \frac{150}{3}\left(\frac{240}{30}\right)^{\frac{2}{3}} = 200, \quad w_{CHN} = \frac{300}{3}\left(\frac{240-K_A}{30-L_A}\right)^{\frac{-1}{3}} = 50
$$

The wage in the US falls and the price of capital increases. In China, the wage increases and the price of capital falls. The fall in wages and the rise in the price of capital is due to the fact that the US now specializes in producing the capital-intensive good where labor is less productive. Similarly in China, wages has increased and the price of capital has decreased because China now specializes in producing the labor-intensive good.
Total number of automobiles produced is 120, and total number of computers is also 120. We can see that total world output has increased precisely because each country is specializing in producing the good it is more suited for.