

12.800: F04 Problem set 6

1) Write Matlab code to model the motion of point vortices. Try to make the code general enough to handle any number of vortices. Provide plots of the trajectories of point vortices for the following cases:

a) Two vortices: vortex one having an initial condition of $(-2.5, 0)$, and vortex two having an initial condition of $(2.5, 0)$. The circulation of vortex one is $\Gamma_1 = -10$, and the circulation of vortex two is $\Gamma_2 = 10$.

b) Two vortices with the same initial locations as in part (a), but with $\Gamma_1 = \Gamma_2 = 10$.

c) Use the method of images to show how two point vortices initially located at $(-2.5, 9)$ and $(2.5, 9)$, with circulations of $\Gamma_1 = -10$ and $\Gamma_2 = 10$ evolve as they approach a solid boundary at $y=0$.

Make the axes of each of the plots extend from ± 10 in both the x and the y directions. Hand in the three plots along with your (well commented) code.

2) In class we proved Kelvin's Circulation Theorem by starting from the expression

$$\Gamma = \oint_c \mathbf{u} \cdot d\mathbf{s}$$

and taking a Lagrangian approach to finding $\frac{d\Gamma}{dt}$. Now prove Kelvin's Circulation Theorem by starting from the expression

$$\Gamma = \iint_A \boldsymbol{\omega} \cdot d\mathbf{A}$$

and taking an Eulerian approach to finding $\frac{d\Gamma}{dt}$. Hint: mimic the approach used to derive the Reynold's Transport Theorem with modifications suitable for the 2D situation here.

3) Show that writing the barotropic, inviscid vorticity equation, $\frac{\partial \boldsymbol{\omega}}{\partial t} + \mathbf{u} \cdot \nabla \boldsymbol{\omega} = \boldsymbol{\omega} \cdot \nabla \mathbf{u}$, in terms of streamfunction gives

$$\frac{D\nabla^2 \psi}{Dt} = 0.$$

What assumptions did you have to make?

4) Provide two questions for a notional midterm exam. The best questions will be included in the final.