12.002 Physics and Chemistry of the Earth and Terrestrial Planets
Fall 2008

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Professors Leigh Royden and Benjamin Weiss

Problem Set #5: Convection
due Friday Oct 24 in class

Problem 1.

Show that the Raleigh number is dimensionless.

$$Ra = d^3\alpha\rho g\Delta T / \mu\kappa$$

κ	thermal diffusivity (m^2/s)
α	coefficient of thermal expansion ($°C^{-1}$)
ΔT	temperature difference across the layer (°C)
μ	viscosity (Pa s = Ns/m^2 = kg/ms)
d	layer thickness and approx. dimension of convection cell (m)
ρ	density (kg/m^3)
g	acceleration due to gravity (m/s^2)

Problem 2.

Show that within the Earth, the net force on even large volumes of the mantle must be zero. You will want to know that the stresses within the mantle are typically of order 1-100 MPa (or 10^6-10^8 Pa), that the maximum velocity for mantle flow appears to be around 100 mm/yr, and that geologic processes operate over time scales of thousands to millions of years. Now consider a cube of mantle 1000 km on a side, with a volume of 10^9 km^3. What velocity is obtained when you apply a tiny net force, equivalent to a stress 10^{-3} MPa, to one face of this mantle cube for a year? What conclusions can you draw from this calculation?

Problem 3.

(a) The steady-state conductive heat flow through a layer of thickness d, with a thermal conductivity K and a temperature difference across the layer of ΔT is just $q_{conductive}=(K\Delta T/d)$. Show that, in the absence of an adiabatic gradient, the purely convective heat flow (that is, the heat flow that we would get if there was no conduction) divided by the purely conductive heat flow is equal to the Rayleigh number times a dimensionless constant.

(b) From this result, describe how convecting layers with the same heat flow through the layer might have very different Rayleigh numbers.

Problem 4.

Suppose that when their solar system was destroyed, aliens of advanced technology moved to a nearby sun and created a planet, identical to their own in internal structures, distributions of heat producing elements, etc. The only difference in is that the new planet has a smaller radius, R, than their original planet, which had a radius R_{earth}=6400 km. However, they also want to be sure that their planetary mantle is convecting, even after it cools to it's steady-state condition, as they have a religious aversion to living on a "dead" planet. What is the smallest radius that they can use to build their new planet with?

You will want to use the following useful formula from the notes (along with the definition of the Raleigh number):

$$\frac{\partial T}{\partial t} = -\beta \frac{d\rho \Delta T^2 g\alpha}{\mu} + \frac{Q_r R}{\rho C_p d}$$

Use the following values in your calculations:

$g = 10 m/s^2 * R/R_{earth}$	$\rho = 4000 kg/m^3$	
$Q_R = 10^{-8} W/m^3$	$C_p = 1260 J/kgK$	$\Delta T = 20°C$
$d = R/2$	$K = 5 W/mK$	$\mu = 10^{21}\ Pa\ s$
$\kappa = 10^{-6} m^2/s$	$\alpha = 10^{-5} °C^{-1}$	$\beta = .01$

Note that g scales linearly with planetary radius, so that g for this planet will simply be the value of g for the earth (10 m/s^2) scaled up or down with planetary radius.

Problem 5.

(a) Using the last equation in the "Cooling Planets" section of your notes and the parameters given at the end of that section, compute for this earthlike planet, using d=R/2, the temperature of the convecting layer and the surface heat flow through time, for 4.5 Ga.

(b) Repeat the computation for a heat production that is lower than in (a) by a factor of 10,000 (in other words, effectively zero).

(c) What can you conclude from comparing the two results? What is buffering the temperature of the convecting system?

$g = 10 m/s^2 * R/R_e$	$C_p = 1260 J/kgK$	$\gamma = .05$
$Q_R = .01 \mu W/m^3 = 10^{-8} W/m^3$	$K = 5 W/mK$	$\mu_o = 10^{22} Pa \cdot s$
$\kappa = 10^{-6} m^2/s$	$\alpha = 10^{-5} K^{-1}$	$T_o = 1300°C$
$\rho = 4000 kg/m^3$	$\beta = .01$	$\Delta T = 10°C$